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M.A.ECONOMICS
(Second Year)

ECONOMETRIC METHODS
(SECM34)

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ECONOMETRIC METHODS

Unit	Details
I	**Econometrics** Econometrics: Meaning – Scope – Methodology – Limitations – Basic Ideas of Linear Regression Model – Two Variable Model – Error Term – Significance - Stochastic vs Nonstochastic Variable.
II	**Regression Analysis** Classical Linear Regression Model – Assumptions – Method of ordinary least square (OLS) – Derivation of OLS – Properties of OLS Estimators – Gauss Markov Theorem – Proof – Multiple Linear Regression Model (Concepts Only)
III	**Multicollinearity** Multicollinearity: Nature – Causes – Consequences – Detection – Remedial Measures
IV	**Auto Correlation** Autocorrelation: Meaning – Nature – Consequences – Detection – Remedial Measures
V	**Heteroscedasticity** Heteroscedasticity: Meaning – Nature – Consequences – Detection – Remedial Measures

Text Books
Jeffrey M Wooldridge, Introductory Econometrics: A Modern Approach, Cengage Learning India Pvt Ltd, New Delhi, 2012.
James H.Stock & Mark W. Watson, Introduction to Econometrics, Pearson Education Pvt. Ltd, Singapore, 2010.
Damodar N. Gujarathi and Sangeetha, Basic Econometrics, Tata McGraw-Hill Publishing Company, New Delhi, 2011.
Koutsoyiannis A, Theory of Econometrics, Palgrave, New York, 2001.
Maddala G. S, Introduction to Econometrics, John Wiley & Sons, Fte. Ltd, Singapore, 2005.

UNIT - I

1.1 Introduction

1.1.1 WHAT IS ECONOMETRICS?

Econometrics refers to the application of economic theory and statistical techniques for the purpose of testing hypothesis and estimating and forecasting economic phenomenon. Literally interpreted, econometrics means "economic measurement." Although measurement is an important part of econometrics, the scope of econometrics is much broader, as can be seen from the following quotations: Econometrics, the result of a certain outlook on the role of economics, consists of the application of mathematical statistics to economic data to lend empirical support to the models constructed by mathematical economics and to obtain numerical results. econometrics may be defined as the quantitative analysis of actual economic phenomena based on the concurrent development of theory and observation, related by appropriate methods of inference. Econometrics may be defined as the social science in which the tools of economic theory, mathematics, and statistical inference are applied to the analysis economic phenomena. Econometrics is concerned with the empirical determination of economic laws.

1.1.2 BASIC ECONOMETRICS

The art of the econometrician consists in finding the set of assumptions that are both sufficiently specific and sufficiently realistic to allow him to take the best possible advantage of the data available to him. Econometricians are a positive help in trying to dispel the poor public image of economics (quantitative or otherwise) as a subject in which empty boxes are opened by assuming the existence of can-openers to reveal contents which any ten economists will interpret in 11 ways. The method of econometric research aims, essentially, at a conjunction of economic theory and actual measurements, using the theory and technique of statistical inference as a bridge pier.

1.2 Objectives

1. It helps to explainthe behavior of a forth coming period that is forecasting economic phenomena.
2. It helps to prove the old and established relationships among the variables or between the variables
3. It helps to establish new theories and new relationships.
4. It helps to test the hypotheses and estimation of the parameter.

Econometric methods are widely used in economic research. Research required different variety of techniques, which is varied from one subject to another. In recent decades an increased emphasis has been laid down on the development and use of statistical techniques for the analysis of the economic problems. **Prof. Ragnar Frisch**,(Father of Econometrics) a Norwegian economist and statistician first of all named this science as "Econometrics" in **1926**. Econometrics emerged as an independent discipline studying economics phenomena. But it recognized and got attention after the world war. In 1931, the realization of the necessity of econometric work had become so evident, which made to form "Econometric Society". This International association includes practically all the worker in the field. The society published a periodical called "*Econometrica*" which disseminates the result of econometric research work. The electronic gadgets like computers have stimulated the utilization of econometrics in recent days.

MEANING AND DEFINITION

a) Meaning

Econometrics means economic measurement. Econometrics deals with the measurement of economic relationships. It"s an amalgamation of economic theory with mathematics and statistics. It is a science which combines economic theory with economic statistics and tries by mathematical and statistical methods to investigate the empirical support of general economic law established by economic theory. The term econometrics is formed from two words of Greek origin, „*oukovouia'* meaning economy and „*uetpov'* meaning measure.

b) Definitions

The book „Econometric Theory" was authored by **Arthur S Goldberger**, and defined econometrics in that book as"Econometrics may be defined as the social science in which the tools of economic theory, mathematics and statistical inference are applied to the analysis of economic phenomena".

Gerhard Tinbergen points out that "Econometrics, as a result of certain outlook on the role of economics, consists of application of mathematical statistics to economic data to lend empirical support to the models constructed by mathematical economics and to obtain numerical results".

H Theil "Econometrics is concerned with the empirical determination of economic laws"

In the words **of Ragnar Frisch** "The mutual penetration of quantitative econometric theory and statistical observation is the essence of econometrics". Thus, econometrics may be considered as the integration of economics, mathematics and statistics for the purpose of providing numerical values for the parameters of economic relationships and verifying economic theories. It is a special type of economic analysis and research in which the general economic theory, formulated in mathematical terms, is combined with empirical measurement of economic phenomena.

SCOPE OF ECONOMETRICS

Scope and areas of application of econometrics is expanding constantly. It includes simple as well as sophisticated mathematical and statistical techniques. Econometrics is the application of specific methods in the general field of economics science. In this sense, it plays a service role to economic analysis. By establishing new relationships and theories it serves the policy makers.

Government Aspect:

Suppose government want to devalue its currency to correct the BOP position. For estimating the consequences of devaluation, the government is concerned with price elasticity"s of imports and exports. The price elasticity is to be estimated with the help of demand function of import and export commodities. Here, the econometric tools will be applied.

Producer Aspect:

Suppose a producer wants to maximize his profit, the producer will choose the level of production which gives him maximum surplus . That is minimum cost of production and maximum output, which will be solved with help of econometric methods. In capitalistic economy too, the econometric help the producers in making rational calculations, Demand function, Price elasticity"s and constraints help a producer to choose his field of investment. Econometrics help in establish new relationships and prove old theorems. Econometrics is the outstanding method for the verification of economic theorem.

Consumer Aspect:

Effect of the taxation on consumers or effects of government expenditures on consumers standard of living are come under the purview of econometric analysis. Optimum allocation of resources has been solved with the development of the theory of programming.

Professor Oscar Lange explained the scope around three groups of questions.

(1) Earlier studies were centered round the main problem of capitalistic economy that is forecasting of business cycle. This type of study was a thing of past.

(2) Secondly econometric researches were connected with market research. Analysis of demand function, Production function, Cost function, Supply function, Distribution of wealth. etc all problems connected with market analysis.

(3) The third group of question related to theory of programming. It includes the questions relating to the whole of the economy. This field is related with planned and socialistic economies. These studies have been stimulated with the growth of communistic countries.

Now – a – days encompass mainly testing hypotheses, estimation of the parameters, usages of estimates of the parameter, ascertaining the proper functional form of economic relations, measuring the effects of imperfect data and study of the feedback relationships. Hence, whatsoever may the part of economy, or types of markets, the econometric tools are very useful for interpreting them. Whether a producer or consumer, supplier or buyer, government or public, econometrics will help in rational calculation in economic phenomena. Econometrics provides equally valuable assistance to normative as well as positive economics.

FEATURES OF AN EQUATION

Econometric theory is mainly concerned with quantitative relationships among economic variables. Quantitative statements are usually expressed in the form of equation with specified numerical coefficients. **Prof. Carl.F.Christ** expressed that the equation must have the following features:

1. An economic equation should be **relevant** to the phenomenon being studied.
2. Equation should be **simpl**e to understand.
3. Equation should be **consistent** and consider only the relevant part of the theory.
4. Equation relating to a problem should be consistent with available **relevant** data.
5. The co-efficient of an equation will affect the economic inferences, so it is desirable to have an **accurate knowledge about the co-efficient.**
6. Equation must have **forecasting ability,** because econometric study concerned with

future.

The above all features can be simplified as follows:

"An equation may have **relevance, simplicity, theoretical probability, explanatory ability, accuracy of co-efficient and forecasting ability**"

TOOLS OF ECONOMETRICS

Tools of Econometrics are Mathematics and Statistics. Econometrics transforms economic theory into mathematical terms and utilizes statistical methods to derive economic relationships under certain assumptions. Algebra, properties of number system, Calculus, Statistical Data, statistical methods of sampling and testing the hypothesis are the tools of Econometrics.

LIMITATIONS OF ECONOMETRICS

a. Applicable only to quantifiable phenomena

b. Lack of moral judgments, possibility of spurious regressions.

c. Irrational human behavior leads challenges in specifying variables and model construction for estimation.

d. Econometric model construction and data analysis are time consuming, tedious and complex because of mathematical statistical knowledge and economic theoretical knowledge are needed.

e. Data are scarce relative to the number of parameter needed to be estimated

f. Econometrics is sometime criticized for relying too heavily on the interpretation of raw data without linking it to establish economic theory or looking for casual machinist

g**.** It tests the hypotheses, but neglects the concerns of error

GOALS OF ECONOMETRICS

The three main goals of econometrics are as follows:

1. Analysis: Econometrics primarily aims at the verification of economic theories. In this case we say that the purpose of the research is analysis. That is**, t**he economic models are formulated in an empirically testable form, to decide how well they explain the observed behavior of the economic units. Several econometric models can be derived from an economic model. Such models differ due to different choice of functional form, specification of stochastic structure of the variables etc. So, a strong analysis will be carried out by econometrics as a prime goal to verify any economic theory and economic phenomena.

2. Policy Making: The models are estimated on the basis of observed set of data and are tested for their suitability. This is the part of statistical inference of the modeling. Various estimation procedures are used to know the numerical values of the unknown parameters of the model. Based on various formulations of statistical models, a suitable and appropriate model is selected. The inference or the knowledge obtain from the numerical value of the coefficients are important for decision making of firms as well as formulation of the economic policy of the government. It helps to compare the effects of alternate policy decision.

3. Forecasting: The obtained models are used for forecasting and policy formulation which is an essential part in any policy decision. Such forecasts help the policy makers to judge the goodness of fitted model and take necessary measures in order to re-adjust the relevant economic variables.

SIGNIFICANCE OF STOCHASTIC DISTURBANCE TERM

The disturbance term is a surrogate for all those variables that are omitted from the model but that collectively affect Y. The reasons for to introduce the stochastic disturbance termUi are as follows:

1. **Vagueness of theory**: The theory, if any, determining the behavior of Y may be, and often is, incomplete. We might know for certain that weekly income X influences weekly consumption expenditure Y, but we might be ignorant or unsure about the other variables affecting Y. Therefore, ui may be used as a substitute for all the excluded or omitted variables from the model.

2.Unavailability of data: Even if we know what some of the excluded variables are and therefore consider a multiple regression rather than a simple regression, we may not have quantitative information about these variables. It is a common experience in empirical analysis that the data we would ideally like to have often are not available. For example, in principle we could introduce family wealth as an explanatory variable in addition to the income variable to explain family consumption expenditure. But unfortunately, information on family wealth generally is not available. Therefore, we may be forced to omit the wealth variable from our model despite its great theoretical relevance in explaining consumption expenditure.

3. **Core variables versus peripheral variables**: Assume in our consumption-income example that besides income X1, the number of children per family X2, sex X3, religion X4, education X5, and geographical region X6 also affect consumption expenditure. But it is quite possible that the joint influence of all or some of these variables may be so small and at best nonsystematic or random that as a practical matter and for cost considerations it does not pay to introduce them into the model explicitly. One hopes that their combined effect can be treated as a random variable ui.

4. **Intrinsic randomness in human behavior**: Even if we succeed in introducing all the relevant variables into the model, there is bound to be some "intrinsic" randomness in individual Y's that cannot be explained no matter how hard we try. The disturbances, the ui‟s, may very well reflect this intrinsic randomness.

5. **Poor proxy variables**: Although the classical regression model assumes that the variables Y and X are measured accurately, in practice the data may be plagued by errors of measurement. Consider, for example, Keynes well-known theory of the Psychological law of consumption function regards consumption expenditure (Yp) as a function of income (Xp). But since data on these variables are not directly observable, in practice we use proxy variables, such as current consumptionexpenditre (Y) and current income (X), which can be observable. Since the observed Y and X may not equal Yp and Xp, there is the problem of errors of measurement. The disturbance term u may in this case then also represent the errors of measurement. As we will see in a later chapter, if there are such errors of measurement, they can have serious implications for estimating the regression coefficients, the p's.

6. **Principle of parsimony**: Following we would like to keep our regression model as simple as possible. If we can explain the behavior of Y "substantially" with two or three explanatory variables and if our theory is not strong enough to suggest what other variables might be included, why introduce more variables? Let ui represent all other variables. Of course, we should not exclude relevant and important variables just to keep the regression model simple

7. **Wrong functional form**: Even if we have theoretically correct variables explaining a phenomenon and even if we can obtain data on these variables, very often we do not know the form of the functional relationship between the regressand and the regressors. Is consumption expenditure a linear (invariable) function of income or a nonlinear (invariable) function? In two-variable models the functional form of the relationship can often be judged from the scatter diagram. But in a multiple regression model, it is not easy to determine the appropriate functional form, for graphically we cannot visualize scatter diagrams in multiple dimensions.

1. Applications of economic theory need a responsible understanding of economic relationships and econometrics method.

2. The econometrics theory thus becomes a very powerful tool for understanding of the applied economic relationships and for meaningful research in economics.

3. In this unit we learn basic theory of econometrics and relevant application of the method.

1.3 Methodology of Econometrics:

Broadly speaking, traditional econometric methodology proceeds along the following lines:

1. Statement of theory or hypothesis.
2. Specification of the mathematical model of the theory
3. Specification of the statistical, or econometric, model
4. Obtaining the data
5. Estimation of the parameters of the econometric model
6. Hypothesis testing
7. Forecasting or prediction
8. Using the model for control or policy purposes.

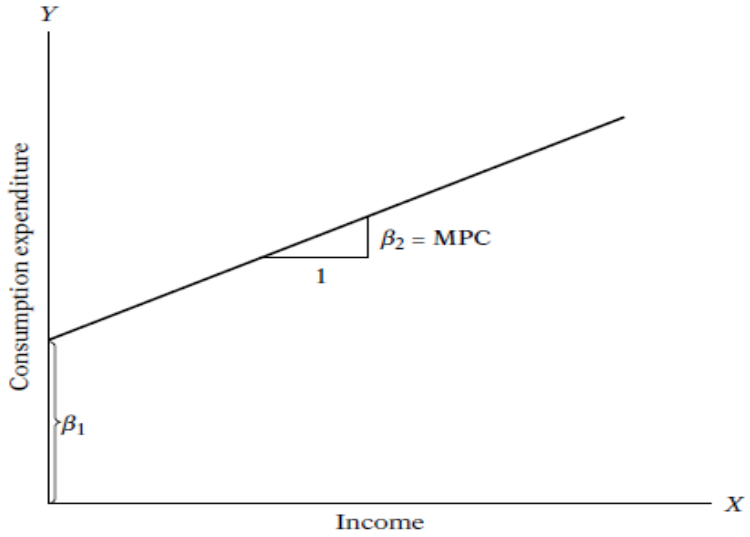
To illustrate the preceding steps, let us consider the well-known Keynesian theory of consumption:

1. Statement of theory or Hypothesis

Keynes postulated that Marginal propensity to consume (MPC), the rate of change of consumption for a unit, change in income, is greater than zero but less than one. i.e., $0 < MPC < 1$

2. Specification of the Mathematical Model of Consumption

Keynes postulated a positive relationship between consumption and income.



Keynesian consumption function.

The slope of the coefficient β_2 measures the MPC.

Keynesian consumption function

$$Y=\beta_1+\beta_2 x \quad O<\beta_2<1$$

Y = Consumption expenditure

X = Income

$\beta_1 x \beta_2$ are knows as the parameters of the model and are respective, the interest and slope of coefficient.

Shows exact and determined relationship between consumption and income.

The slope of the coefficient β_2, measures the MPC.

Equation states that consumption is linearly related to income (Example of a mathematical model of the relationship between consumption and income that is called consumption function in economic).

Single or one equation is known as single equation model and more than one equation is known as multiple equation model.

3. Specification of the econometric model of consumption.

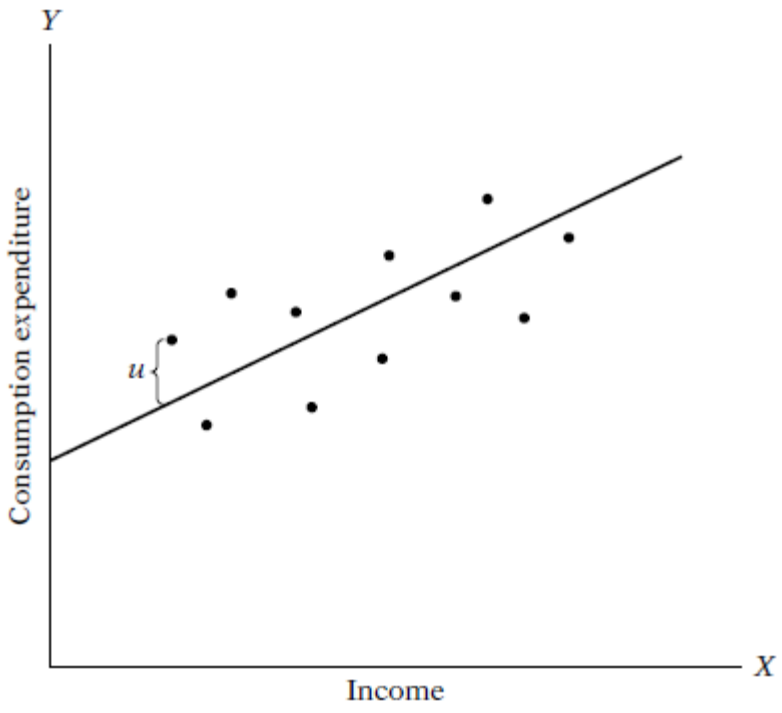
The inexact relationship between economic variables, the econometrician would modify the deterministic consumption function as.

$$Y=\beta_1+\beta_2 x+U$$

This equation is an example of the econometric model. More technically, it is an ex. of linear regression model.

This you may be well represent all those factors that affect consumption but are not taken into account explicitly.

The econometric consumption function hypothesizes that the dependent variable Y (consumption) is linearly related to the explanatory variable X (Income) but that is the relationship between. The two is not exact, it is subject to individual variation.



Econometric model of the Keynesian consumption function.

Q: Why inexact (not exact) relationship exits?

A: Because in addition to income, other variables affect consumption expenditure. For ex. are of family, ages of members of family, religion etc are likely to exert some influence on consumption.

4. Original Data

To obtain the numerical values of $\beta_1 \& \beta_2$ we need data.

{PCE $\rightarrow$ Personal consumption expenditure)

Y variable in this table is the aggregate PCE &xis GD. a measure of aggregate income.

Note: MPC: Average change in consumption over to change in real income.

5. Estimation of the Econometric Model

The statistical technique of regression analysis is the main tool used to obtain the estimates.

The estimated consumption function

$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 x_i$

$\hat{Y}$ = Estimate of Y The estimated consumption function (i.e., regress line).

Regression Analysis is used to obtain estimates.

6. Hypothesis Testing:

Keynes expected the MPC is positive but less than 1.

Confirmation or refulation of economic theories on the basis of sample evidence is based on a branch of statistical theory known as statistical inference (hypothesis testing)

7. Forecasting or Prediction

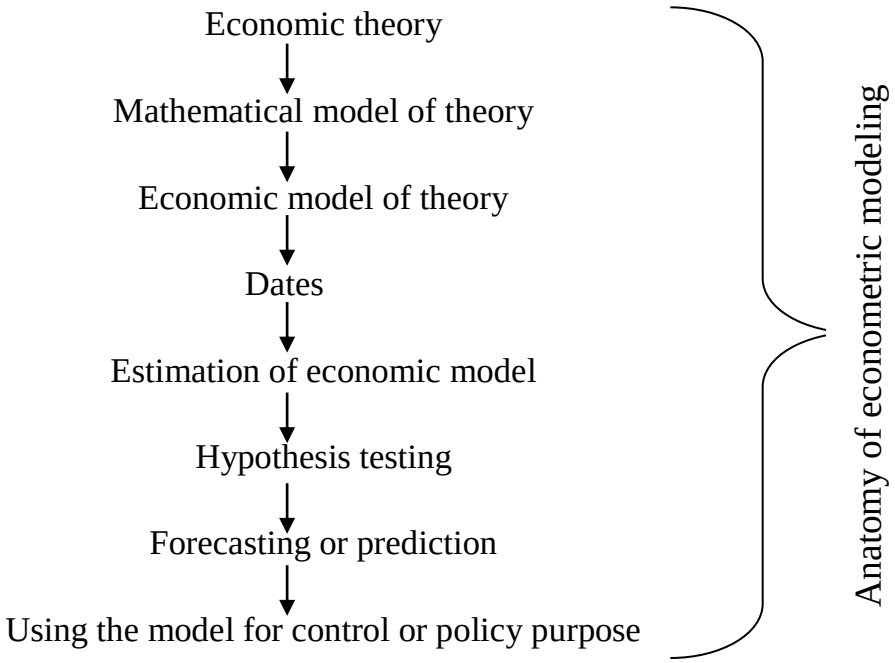
If the chosen model does refute the hypothesis or theory under consideration, we may use it to predict the future value(s) of the dependent, or forecast, variable Y on the basis of known or expected future value(s) of the explanatory, or predictor variable X.

Macroeconomic theory shows, the change in income following change in investment expenditure is given by the income multiplier M.

$$M=\frac{1}{1-MP(}$$

The quantitative estimate of MPC provider valuable information for policy purposes knowing MPC, one can predict the future course of income, consumption expenditure, and employment following a change in the government's fiscal policies.

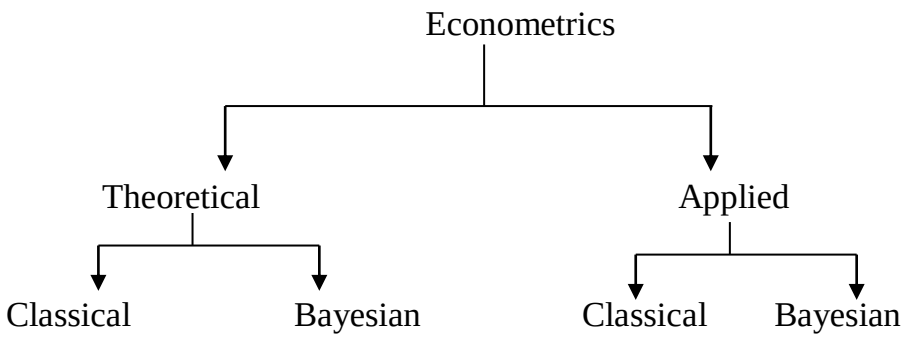
8. Use of the Model for control or Policy purpose



Note:

- Milton Friedmen has developed a model of consumption theory permanent income hypothesis.
- Robert Hall has developed a model of consumption as life cycle permanent income hypothesis

1.4 Types of Econometrics



- Theoretical econ is concerned with the development of appropriate methods of measuring economic relationship specified by economic models.
- Applied econ uses the tool of theoretical econ to study some special fields of eco and business, such as production function etc.

1.5 SUMMARY AND CONCLUSIONS:

Econometrics is an amalgam of economic theory, mathematical economics, economic statistics, and mathematical statistics. Yet the subject deserves to be studied in its own right for the following reasons.

Economic theory makes statements or hypotheses that are mostly qualitative in nature. For example, microeconomic theory states that, other things remaining the same, a reduction in the price of a commodity is expected to increase the quantity demanded of that commodity. Thus, economic theory postulates a negative or inverse relationship between the price and quantity demanded of a commodity. But the theory itself does not provide any numerical measure of the relationship between the two; that is, it does not tell by how much

the quantity will go up or down as a result of a certain change in the price of the commodity. It is the job of the econometrician to provide such numerical estimates. Stated differently, econometrics gives empirical content to most economic theory.

The main concern of mathematical economics is to express economic theory in mathematical form (equations) without regard to measurability or empirical verification of the theory. Econometrics, as noted previously, is mainly interested in the empirical verification of economic theory. As we shall see, then econometrician often uses the mathematical equations proposed by the mathematical economist but puts these equations in such a form that they lend themselves to empirical testing. And this conversion of mathematical into econometric equations requires a great deal of ingenuity and practical skill.

1.6 LETS SUM IT UP :

In last ,we can say that the subject of econometrics deals with the economic measurement . And further, it is defined as the social science in which the tools of economic theory, mathematics, and statistical inference are applied to the analysis of economic phenomena. It is also concerned with the empirical determination of economic laws

1.7 EXCERCISES:

Q.1What do mean by Econometrics?

Q.2 Explain the various steps involved in the methodology of Econometrics?

Q.3What are the various types of Econometrics?

Q.4 How Econometrics can be used as a tool for forcasting and prediction?

Q.5What is Theoretical Econometrics?

Q.6 What is Applied Econometrics?

1.8 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

UNIT - II

2.1 Introduction:

The term *regression* was introduced by Francis Galton. In a famous paper, Galton found that, although there was a tendency for tall parents to have tall children and for short parents to have short children, the average height of children born of parents of a given height tended to move or "regress" toward the average height in the population as a whole.1 In other words, the height of the children of unusually tall or unusually short parents tends to move toward the average height of the population. Galton's *law of universal regression* was confirmed by his friend Karl Pearson, who collected more than a thousand records of heights of members of family groups.2 He found that the average height of sons of a group of tall fathers was less than their fathers' height and the average height of sons of a group of short fathers was greater than their fathers' height, thus "regressing" tall and short sons alike toward the average height of all men. In the words of Galton, this was "regression to mediocrity."

2.1.1 THE MODERN INTERPRETATION OF REGRESSION

The modern interpretation of regression is, however, quite different. Broadly speaking, we may say Regression analysis is concerned with the study of the dependence of one variable,the *dependent variable*, on one or more other variables, the *explanatory variables*,with a view to estimating and/or predicting the (population) mean or average value of the former in terms of the known or fixed (in repeated sampling)
values of the latter.

2.2 Objectives:

1. The key objective behind regression analysis is the statistical dependence of one variable, the dependent variable, on one or more other variables, the explanatory variables.

2. The objective of such analysis is to estimate and/or predict the mean or average value of the dependent variable on the basis of the known or fixed values of the explanatory variables.

3. In practice the success of regression analysis depends on the availability of the appropriate data.

4. In any research, the researcher should clearly state the sources of the data used in the analysis, their definitions, their methods of collection, and any gaps or omissions in the data as well as any revisions in the data.

5. The data used by the researcher are properly gathered and that the computations and analysis are correct.

2.3 WHAT IS REGRESSION ANALYSIS:

Under single regression model one variable, called the dependent variable is expressed as a linear function of one or more other variable, called explanatory variable.

2.3.1 TWO VARIABLE REGRESSION MODEL ANALYSIS:

That means a function has only one dependent variable and only one independent variable.

Two variable or bivariate

Means regression in which the dependent variable (the regressand) is related to a single explanatory variable (the regression).

When mean values depend upon conditioning (variable X) is called conditional expected value. Regression analysis is largely concerned with estimating and/or predicting the (population) mean value of the dependent variable on the basis of the known or fixed values of the explanatory variable (s).

WEEKLY FAMILY INCOME X, $

Y ↓ \ X →	80	100	120	140	160	180	200	220	240	260
Weekly family consumption expenditure Y, $	55	65	79	80	102	110	120	135	137	150
	60	70	84	93	107	115	136	137	145	152
	65	74	90	95	110	120	140	140	155	175
	70	80	94	103	116	130	144	152	165	178
	75	85	98	108	118	135	145	157	175	180
	–	88	–	113	125	140	–	160	189	185
	–	–	–	115	–	–	–	162	–	191
Total	325	462	445	707	678	750	685	1043	966	1211
Conditional means of Y, $E(Y \mid X)$	65	77	89	101	113	125	137	149	161	173

To understand this, consider the data given in the below table. The data in the table refer to a total population of 60 families in a hypothetical community & their weekly income (X) and weekly consumption expenditure (Y), both in dollars. The 60 families are divided into 10 income groups (from $80 to $260) and the weekly expenditures of each family in the various groups are as shown in the table. Therefore, we have 10 fixed values of X and the corresponding Y values against each of the X values; and hence there are 10 Y subpopulations. There is considerable variation in weekly consumption expenditure in each income group, which can be seen clearly but the general picture that one gets is that, despite the variability of weekly consumption expenditure within each income bracket, on the average, weekly consumption expenditure increases as income increases. To see this clearly, in the given table we have given the mean, or average, weekly consumption expenditure corresponding to each of the 10 levels of income. Thus, corresponding to the weekly income level of $80, the mean consumption expenditure is $65, while corresponding to the income level of $200, it is $137. In all we have 10 mean values for the 10 subpopulations of Y. We call these mean values conditional expected values, as they depend on the given values of the (conditioning) variable X. Symbolically, we denote them as $E(Y \mid X)$, which is read as the expected value of Y given the value of X.

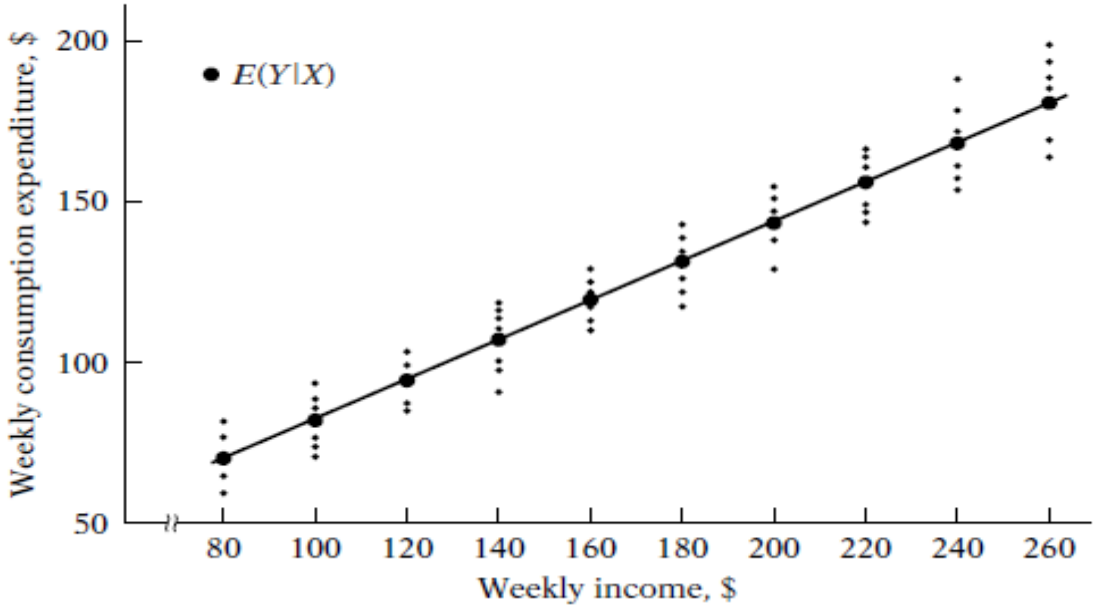


fig.: Conditional distribution of expenditure for various levels of income

It is important to distinguish these conditional expected values from the unconditional expected value of weekly consumption expenditure, $E(Y)$. If we add the weekly consumption expenditures for all the 60 families in the population and divide this number by 60, we get the number \$121.20 (\$7272/60), which is the unconditional mean, or expected, value of weekly consumption expenditure, $E(Y)$; it is unconditional in the sense that in arriving at this number we have disregarded the income levels of the various families. Obviously, the various conditional expected values of Y given in given table are different from the unconditional expected value of Y of \$121.20. When we ask the question, "What is the expected value of weekly consumption expenditure of a family," we get the answer \$121.20 (the unconditional mean). But if we ask the question, "What is the expected value of weekly consumption expenditure of a family whose monthly income is, differently, if we ask the question, "What is the best (mean) prediction of weekly expenditure of families with a weekly income of \$140," the answer would be \$101. Thus the knowledge of the income level may enable us to better predict the mean value of consumption expenditure than if we do not have that knowledge. This probably is the essence of regression analysis, as we shall discover throughout this text.

The dark circled points in figure show the conditional mean values of Y against the various X values. If we join these conditional mean values, we obtain what is known as the population regression line (PRL), or more generally, the population regression curve**.** More simply, it is the regression of Y on X. The adjective "population" comes from the fact that we are dealing in this example with the entire population of 60 families. Of course, in reality a population may have many families.

Geometrically, then, a population regression curve is simply the locus of the conditional means of the dependent variable for the fixed values of the explanatory variable(s). More simply, it is the curve connecting the means of the subpopulations of Y corresponding to the given values of the regressor X. It can be depicted as in figure.

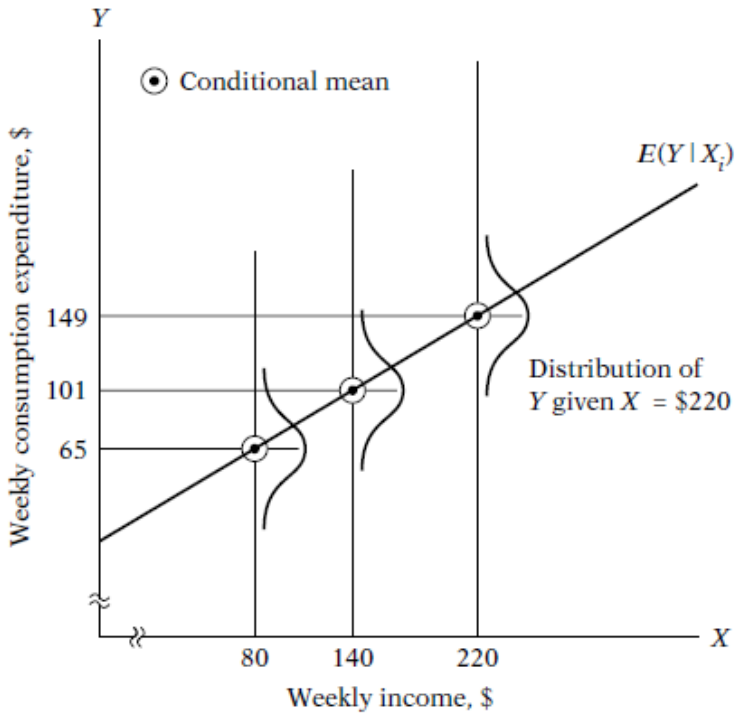


Fig.: Population Regression line.

This figure shows that for each X (i.e., income level) there is a population of Y values (weekly consumption expenditures) that are spread around the (conditional) mean of those Y values.

For simplicity, we are assuming that these *Y* values are distributed symmetrically around their respective (conditional) mean values. And the regression line (or curve) passes through these (conditional) mean values.

Concept of Population Regression function (PRF) Or Conditional Expectation function

$$\Sigma(Y/X_i)=f(x_i)$$

$f(X_i)$: Some function of the explanatory variable X

$\Sigma(Y/X_i)$: Linear function of X_i

$$\Sigma(Y/X_i)=\beta_1+\beta_2 X_i$$

$\beta_1 \& \beta_2$ are unknown but fixed parameters known as the regression coefficients are also known as intercept and slope coefficient.

In regression analysis our interest is in estimating the PRFs.

2.3.2 ESTIMATION THROUGH OLS

Properties of OLS:

1) Our estimation are expressed solely in term of observatory can be easily complete.

2) They are point estimation.

3) Once OLS estimation is obtained from the sample data. The sample regression line can be easily obtained.

$$Y_i=(b_0+b_1 x_{1i}+b_2 x_{2i})+(u_i)$$

Assumptions of Model

1) Variable u is real random variable.

2) Homoscedasticity

$$E(u_1^2)=\sigma^2$$

3) Normality of u

$$u\sim N(O,\sigma_0^2)$$

4) Non auto correlation

$$E(u_i u_j)=u \quad i\neq j$$

5) Zero mean of u

$$E(u_i)=0$$

6) Independence of u_i and X_i.

$$E(u_i/x_{1i})=E(u_i X_{2i})=0$$

7) No perfect multicollinear X's

8) No error of measurement in the X's.

Estimation through OLS

$$\hat{Y}_i=\hat{\beta}_1+\hat{\beta}_2 X_i+\hat{u}$$

$$Y_i=\hat{Y}+\hat{u} \qquad (Y_i-\hat{u}_i=\hat{Y}_i)$$
$$\hat{u}_i=Y_i-\hat{Y}_i \qquad (\hat{\beta}_1+\hat{\beta}_2 x_i+\hat{u}_i-\hat{u}_i=\hat{Y})$$
$$\hat{u}_i=Y_i-\hat{\beta}_1-\hat{\beta}_2 X_i \qquad (\hat{Y}_i=\hat{\beta}_1+\hat{\beta}_2 X_i)$$

$$\therefore \quad \sum\hat{u}_i^2=Y_i-\hat{Y}$$

Sq. them we get variation of deviation

$\hat{u} = (Y_i - \hat{Y}_i)^2$

$\sum \hat{u}_i^2 = (Y_i - \hat{Y}_i)^2$

$\sum \hat{u}_i^2 = \Sigma(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_1)^2$

$\frac{\delta \sum \hat{u}_i^2}{\delta \beta_1} = 2\Sigma(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_1) = 0$

$\Sigma Y_i = \Sigma(\hat{\beta}_1 - \hat{\beta}_2 X_1)$

$\Sigma Y_i = n\hat{\beta}_1 - \hat{\beta}_2 \sum X_1$ n= sample size

$\frac{\delta \sum \hat{u}_i^2}{\delta \beta_2} = 2\Sigma(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_1)(X_i) = 0$

$X_i \Sigma(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_1) = 0$

$\Sigma X_i Y_i = X_i \Sigma(\hat{\beta}_1 - \hat{\beta}_2 X_1)$

$\Sigma X_i Y_i = \hat{\beta}_1 X_i - \hat{\beta}_2 X_1^2$

Note:- We are not taking n β_2 because one variable X_1 is already percent. So no need for n, co_2 they are one & the same.

(LRM) = Classical linear regression Modes) Normal equation Y is dependent upon X. X is independent.)

Q. Find the value of $\hat{\beta}_1 \& \hat{\beta}_2$

$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_i \quad \rightarrow \quad (1)$

$$\sum Y_i = n\hat{\beta}_1 + \hat{\beta}_2 \sum X_i \qquad \rightarrow \quad (2)$$

$$\sum X_i Y_i = \hat{\beta}_1 \sum X_i + \hat{\beta}_2 \sum X_i^2 \rightarrow \quad (3)$$

Dividing equator (2) by n

$$\frac{\sum Y_i}{n} = \frac{n\hat{\beta}_1}{n} + \frac{\hat{\beta}_2 \sum X_i}{n}$$

$$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$$

$$\hat{\beta}_1 = \hat{\beta}_2 X - \bar{Y}$$

Now after further simplification we get the value of $\hat{\beta}_2$ as

$$\hat{\beta}_2 = \frac{\sum xy}{\sum X_i^2}$$

2.4 SUMMARY AND CONCLUSIONS:

1. The key concept underlying regression analysis is the concept of the **conditional expectation function (CEF), or population regression function (PRF).** Our objective in regression analysis is to find out how the average value of the dependent variable (or regressand) varies with the given value of the explanatory variable (or regressor).

2. This lesson largely deals with **linear PRFs,** that is, regressions that are linear in the parameters. They may or may not be linear in the regressand or the regressors.

3. For empirical purposes, it is the **stochastic PRF** that matters. The **stochastic disturbance term** *ui* plays a critical role in estimating the PRF.

4. The PRF is an idealized concept, since in practice one rarely has access to the entire population of interest. Usually, one has a sample of observations from the population. Therefore, one uses the **stochastic sample regression function (SRF)** to estimate the PRF.

2.5 Lets sum it up:

In the concluding remarks, we can say that regression analysis is concerned with the study of the dependence of one variable, the *dependent variable*, on one or more other variables, the *explanatory variables*, with a view to estimating and/or predicting the (population) mean or average value of the former in terms of the known or fixed (in repeated sampling values of the latter. If we are studying the dependence of a variable on only a single explanatory variable, such as that of consumption expenditure on real income, such a study is known as *simple*, or **two-variable, regression analysis.** However, if we are studying the dependence of one variable on more than one explanatory variable, as in the crop-yield, rainfall, temperature, sunshine, and fertilizer examples, it is known as **multiple regression analysis.** In other words, in two-variable regression there is only one explanatory variable, whereas in multiple regression there is more than one explanatory variable.

2.6 Excercise:

1. What is the conditional expectation function or the population regression function?

2. What is the difference between the population and sample regression functions? Is this a distinction without difference?

3. What is the role of the stochastic error term *ui* in regression analysis? What is the difference between the stochastic error term and the residual, $\hat{u}i$?

4. Why do we need regression analysis? Why not simply use the mean value of the regressand as its best value?

5. What do we mean by a *linear* regression model?

2.7 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

4. Green, W.(2000). Econometrics, Prentice Hall of India, New Delhi.

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PROPERTIES OF LEAST - SQUARES ESTIMATOR

3.1 Introduction:

To estimate the population regression function (PRF) on the basis of the sample regression function (SRF) as accurately as possible, we will discuss two generally used methods of estimation:

(1) **Ordinary least squares (OLS)** and

(2) **Maximum likelihood (ML).**

By and large, it is the method of OLS that is used extensively in regression analysis primarily because it is intuitively appealing and mathematically much simpler than the method of maximum likelihood. Besides, as we will show later, in the linear regression context the two methods generally give similar results.

3.2 Objectives:

1. The key objective is to find the the least-squares estimators, in the class of unbiased linear estimators, have minimum variance, that is, they are BLUE.

2. The **goodness of fit** of the fitted regression line to a set of data; that is, we shall find out how "well" the sample regression line fits the data .

3.3 Gauss-Markov Theorem/Blue:

The least-squares estimates possess some ideal or optimum properties, these properties are contained in the well-known **Gauss–Markov theorem.** To understand this theorem, we need to consider the **best linear unbiasedness property** of an estimator.

BLUE: - Best Linear-Unbiased Estimator.

MVUE: - Minimum Variance unbiased Estimator.

- If in BLUE, L is not there, because Linearity in co-effects are required not in X &Y.

The properties if Least-Square are known as the BLUE.

3.3.1 Properties

1. It is linear i.e. a linear function of a random variable such as the dependent variable Y in the regression model.

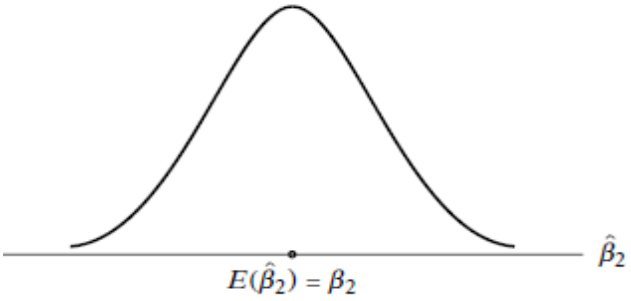
2. It is unbiased i.e its average value, $E(\hat{\beta}_2)$, is = true value of β_2.

3. Has minimum variance in class of all linear unbiased estimators.

(Note:- An unbiased estimator with the least variance is known as an efficient variable.)

3.3.2 Gauss Theorm:- Give the assumption of the classical linear regression Model the least squares estimators; in the class of unbiased linear estimator have minimum variance, that is they are BLUE.

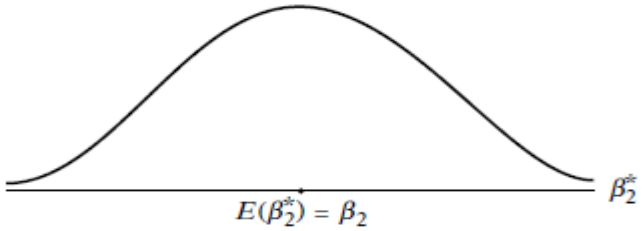
a) The mean of the $\hat{\beta}_2$values. $E(\hat{\beta}_2)$is equal to the true value of β_2. $\hat{\beta}_2$is an unbiased estimator.



(a) Sampling distribution of β_2

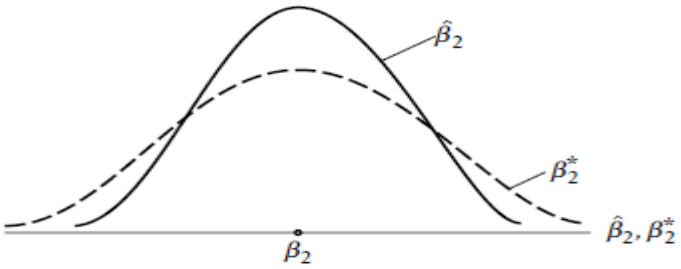
b)

- Sample distribution of β_2, an alternative estimator of β_2.
- $\hat{\beta}_2$& β_2^*. are linear estimators that is they are linear function of Y.
- β_2^* like β_2 is unbiased that is, its average or expected value is equal to β_2.



(*b*) Sampling distribution of β_2^*

c) The variance of β_2^* is larger than the variance of $\hat{\beta}_2$.One would choose the BLUE estimator



(*c*) Sampling distributions of β_2 and β_2^*

G.M. Theorem makes no assumption about the probability distribution of the random variable u_i and therefore of Y_i.

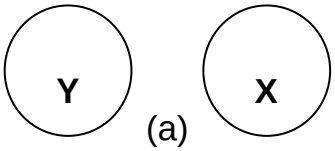
- As long as the assumption of CLRM are satisfied, the theorem holds.
- If any of the assumption doesn't hold, the theoram is invalid.

3.4 Derivation of R^2

Coefficient of determination (r^2).

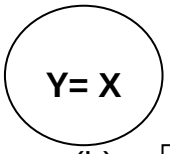
A measure of "Goodness of fit"

- Goodness to fit of the fitted regression line fits the data; that is we shall find out how will the sample regression line fits the data.
- The coefficient of determination r^2 (Two variable case) or R^2 (multiple regression) is a sum many measure that tells how will the sample regression line fits the data.



(a)

$r^2 = 0$



(b)

$r^2 = 1$

% 100 of variation in Y is explanatory by X)

Y = Dependent variable

X= Explanatory variable

Greater the extent of the overlap, the greater the variance in Y is explained by X. r^2 simply a numerical measure of this overlap.

r^2 computation

$$Y_i = \hat{Y}_1 + \hat{u},$$

in the derivation form

$$y_i = \hat{y}_1 + \hat{u}$$

Squaring both side.

$$\sum y_i^2 = \sum(\hat{y}, + \hat{u})^2$$
$$\sum y_i^2 = \sum(\hat{y}_i^2 + \sum \hat{u}_i^2 + 2\sum \hat{y}, \hat{u})$$
$$\sum y^2 = \hat{\beta}_2^2 \sum X_i + \sum \hat{u}^2$$

$$\sum \hat{y}_i \hat{u}_i = 0$$
$$\sum \hat{y}_i = \hat{\beta}_2 X_i$$

TSS = ESS + RSS

Where a) TSS = Total sum of squares.

i.e. $Ey^2 = \Sigma(Y_i - \overline{Y})^2$

b) ESS =Estimated sum of squares.

i.e. $$E\hat{Y}_1^2 = E(\hat{Y}_i - \hat{Y})^2 = E(\hat{y} - y_+)^2 = \hat{\beta}_2^2 \sum X_i^2$$

c) RSS= Residual sum of squares.

i.e. $E\hat{u}_1^2$

Dividing between by TSS $\frac{\text{TSS}}{\text{TSS}} = \frac{\text{ESS}}{\text{TSS}} + \frac{RSS}{TSS}$

$$1 = \frac{\sum(\hat{Y}_1 - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} + \frac{\sum \hat{u}_1^2}{\sum(Y_i - \bar{Y})^2}$$

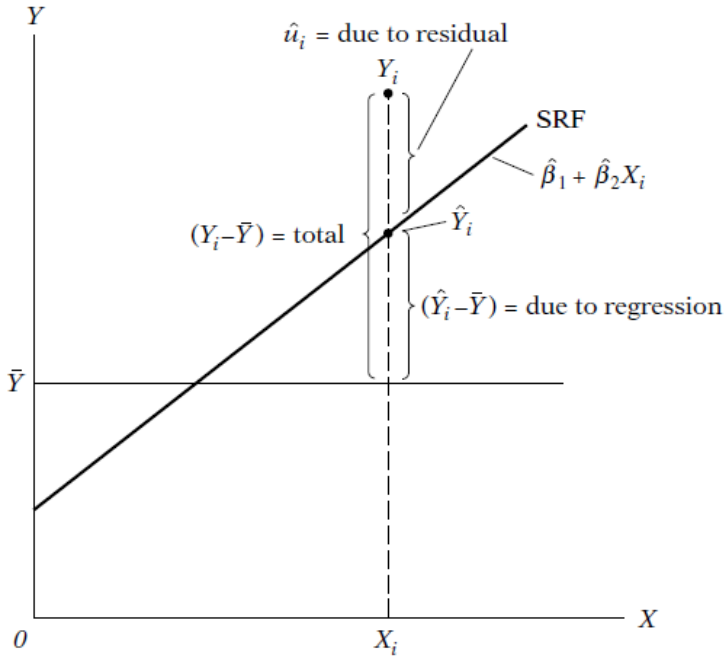
$$1 = \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} = \frac{\sum \hat{u}_i}{\sum(Y_i - \bar{Y})^2}$$

$$\left\lfloor r^2 = \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} \right\rfloor$$

$1 - r^2 = \frac{\text{RSS}}{\text{TSS}}$

$r^2 = 1 - \frac{\text{RSS}}{\text{TSS}}$

r^2 thus defined is known as the (sample) coefficient of determination and is the most commonly used measure of goodness of fit.



Breakdown of the variation of Y_i into two components.

r^2 measure the proportion or % of the two variable in Y explained by regression model.

3.4.1 Two properties of r^2

1. It is a non negative quantity.

2. Its limits are $0 \leq r^2 \leq 1$.

An r^2 1 means a perfect fit r^2 of 0 means no relation.

A quantity closely related to but conceptually very much different from r 2 is the coefficient of correlation, is a measure of the degree of association between two variables. It can be computed from

$$r = \pm\sqrt{r\ 2}$$

Some of the properties of r are as follows:

1. It can be positive or negative, the sign depending on the sign of the term in the numerator of, which measures the sample covariation of two variables.

2. It lies between the limits of −1 and +1; that is, $-1 \leq r \leq 1$.

3. It is symmetrical in nature; that is, the coefficient of correlation between *X* and *Y*(*rXY*) is the same as that between *Y* and *X*(*rYX*).

4. It is independent of the origin and scale; that is, if we define $X^*i = aXi + C$ and $Y^*i = bYi + d$, where $a > 0$, $b > 0$, and *c* and *d* are constants, then *r* between *X** and *Y** is the same as that between the original variables*X* and *Y*.

5. If *X* and *Y* are statistically independent the correlation coefficient between them is zero; but if $r = 0$, it does not mean that two variables are independent. In other words, **zero** correlation does not necessarily imply independence.

6. It is a measure of linear association or linear dependence only; it has no meaning for describing nonlinear relations.

3.5 SUMMARY AND CONCLUSIONS:

The important topics and concepts developed in this lesson can be summarized as follows.

1. Based on these assumptions, the least-squares estimators take on certain properties summarized in the Gauss–Markov theorem, which states that in the class of linear unbiased estimators, the least-squares estimators have minimum variance. In short, they are BLUE.

2. The precision of OLS estimators is measured by their standard errors.

3. The overall goodness of fit of the regression model is measured by the coefficient of determination, r 2. It tells what proportion of the variation in the dependent variable, or regressand, is explained by the explanatory variable, or regressor. This r 2 lies between 0 and 1; the closer it is to 1, the better is the fit.

4. A concept related to the coefficient of determination is the coefficient of correlation, r. It is a measure of linear association between two variables and it lies between −1 and +1.

3.6 LETS SUM IT UP:

In last we can say that to find the the least-squares estimators, in the class of unbiased linear estimators, have minimum variance, that is, they are BLUE we can use the Gauss–Markov theorem and the coefficient of determination r 2 (two-variable case) or R2 (multiple regression) is a summary measure that tells how well the sample regression line fits the data. The coefficient of determination helps in finding the goodness of fit of the fitted regression line to a set of data; that is, we shall find out how "well" the sample regression line fits the data.

3.7 EXCERCISES:

Q.1 State and prove Gauss Markov theorem.

Q.2 Discuss in detail the difference between PRF and SRF.

Q.3 Discuss adjusted R square.

Q.4 What is non spherical error term?

Q.5 Does it matter if we regress X on Y or Y on X.

Q.6 Write a short note on statistical versus deterministic relationship.

Q.7 Prove maximum likelihood estimation of multiple regression model and find out why ML estimator is biased.

Q.8 In two variable case, first derive the normal equations and then from them find out the values of $\beta 1$ and $\beta 2$?

3.8 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

4. Green, W.(2000). Econometrics, Prentice Hall of India, New Delhi.

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THE CLASSICAL LINEAR REGRESSION MODEL

4.1 INTRODUCTION:

If our objective is to estimate β1 and β2 only, the method of OLS will be suffice. But in regression analysis our objective is not only to obtain ˆ β1 and ˆ β2 but also to draw inferences about the true β1 and β2. For example, we would like to know how close ˆ β1 and ˆ β2 are to their counterparts in the population or how close ˆYi is to the true E(Y | Xi). To that end, we must not only specify the functional form of the model, but also make certain assumptions about the manner in which Yi are generated. To see why this requirement is needed, look at the PRF: Yi = β1 + β2Xi + ui . It shows that Yi depends on both Xi and ui . Therefore, unless we are specific about how Xi and ui are created or generated, there is no way we can make any statistical inference In this lesson, we will study about the various methods through which the regression models draw inferences about the various parameters. Basically, there are three methods through which we do this:-

1. The classical linear regression model (CLRM).
2. Generalized least square (GLS).
3. Maximum Likelihood estimation (ML)

4.2 OBJECTIVES:

1. In regression analysis our objective is not only to obtain $\hat{\beta}_1$ and $\hat{\beta}_2$ but also to draw inferences about the true β_1 and β_2. For example, we would like to know how close $\hat{\beta}_1$ and $\hat{\beta}_2$ are to their counterparts in the population or how close $\hat{Y}_i$ is to the true $E(Y \mid X_i)$.

2. Look at the PRF: $Y_i = \beta_1 + \beta_2 X_i + u_i$. It shows that Y_i depends on both X_i and u_i . The assumptions made about the X_i variable(s) and the error term are extremely critical to the valid interpretation of the regression estimates.

3. Our objective is to first discuss the assumptions in the context of the two-variable regression model,we extend them to multiple regression models, that is, models in which there is more than one regressor.

4.3 THE CLASSICAL LINEAR REGRESSION MODEL:

The assumptions underlying the method of least squares

The Gaussian, standard, or classical linear regression model (CLRM), which is the cornerstone of most econometric theory, makes 10 assumption.

Assumption 1: Linear regression model. The regression model is linear in the parameters,

$Y_i = \beta_1 + \beta_2 X_i + u_i$

Assumption 2: **X values are fixed in repeated sampling**. Values taken by the regressor X are considered fixed in repeated samples. More technically, X is assumed to be nonstochastic.

Assumption 3: Zero mean value of disturbance u_i. Given the value of X, the mean, or expected, value of the random disturbance term u_i is zero. Technically, the conditional mean value of u_i, is zero. Symbolically, we have

$E(ui\ |X_i) = 0$

Assumption 4: Homoscedasticity or equal variance of u_i. Given the value of X, the variance of u_i is the same for all observations. That is the conditional variance of u_i, are identical. Symbolically, we have

$$\text{var}\ (u_i\ |X_i) = E(u_i\ |X_i)^2$$

$$= E(u_i^2\ |X_i) \text{ because of Assumption 3}$$

$$= \sigma^2$$

Where var stands for variance

Assumption 5: No autocorrelation between the disturbances. Given any two X values, X_i and X_j (i≠j) the correlation between any two u_i and u_j (i≠j) is zero. Symbolically

$$\text{Cov}(u_i\, u_i \,|X_i, X_j) = \text{E}\{[u_i - E(u_j)]|\, X_i\}\ \{[u_i - E(u_j)]|\ X_i)$$

$$= \text{E}(u_i\,|\text{X}_\text{i})\ (u_j\,|\text{X}_\text{j})$$

$$= 0$$

Where i and j are two different observation and where cov means covariance.

Assumption 6: Zero covariance between u_i and X_i or $E(u_iX_i) = 0$ Formally,

$$\text{Cov}(u_i\, X_i) = \text{E}[u_i - E(u_j)][X_i - E(x_I)]$$

$$= \text{E}[u_i\,(\text{X}_\text{i} - \text{E}(\text{X}_\text{i}))] \text{ Since E}(u_i) = 0$$

$$= \text{E}[u_i\text{X}_\text{i}) - \text{E}(\text{X}_\text{i})\ \text{E}(u_i) \text{ Since E}(X_i) \text{ is nonstochastic}$$

$$= \text{E}[u_i\text{X}_\text{i}) \text{ Since E}(\text{u}_i) = 0$$

$$= 0 \text{ by assumption}$$

Assumption 7: The number of observation *n* must be greater than the number of parameters to be estimated. Alternatively, the number of observation n must be greater than the number of explanatory variables.

Assumption 8: Variability in X values. The X values in a given sample and not all be the same. Technically, var (X) must be a finite positive number.

Assumption 9: The regression model is correctly model in correctly specified. Alternatively, there is no specification bias or error in the model used in empirical analysis.

Assumption 10: There is no perfect multicolinearity. That is, there are no perfect linear relationships among the explanatory variable.

4.10 GENERALISED LEAST SQUARE (GLS)

OLS method doesn't follow this strategy & therefore doesn't make use of the information contained in the unequal variability of the dependent variable Y.

But GLS takes such information into accent explicitly & is therefore capable of producing estimators that are BLUE.

$$Y_i = \beta_1 + \beta_2 X_i + u \qquad \rightarrow \quad (1)$$

Which for case of algebraic manipulation

$$Y_i = \beta_1 X_{oi} + \beta_2 X_i + u_i \qquad \rightarrow \quad (2) \qquad X_{0i}=1$$

$$\frac{Y_i}{\sigma_i} = \beta_1\left(\frac{X_{oi}}{\sigma_i}\right) + \beta_2\left(\frac{X_i}{\sigma_i}\right)\left(\frac{u_i}{\sigma_i}\right) \qquad \rightarrow \quad (3) \qquad \text{for each i}$$

$$Y_i^* = \beta_1^* X_{oi}^* + \beta_2^* X_i^* + u_i^* \qquad \rightarrow \quad (4)$$

{Where transformed, variable are that are divided by σ_i}. We use the notation.

$\sigma^2 \rightarrow$ heteroscedastic variable

What is the purpose of transforming the original mode?

Notice the following feature of the transformed error term u_i^*

Var ($u_i^* = \sum (u_i^*)^2 = \sum \frac{(u_o)^2}{\sigma_1}$

= $\frac{1}{\sigma_1^2} \sum (u_1^2)$ {σ^2 is known)

= $\frac{1}{\sigma_1^2} (\sigma_1^2)$ $\Sigma(\mu_1^2) = \sigma_1^2$

This procedure of transforming original variable in such a way that the transformed variable satisfy the assumption of the classical model & then apply OLS to then is known as the method of GLS.

In short GLS is OLS on the transformed variables that satisfy the standard last sq. assumption.

4.11 Maximum Likelihood estimation (ML)

Assumption 2 variable modes

$Y_1 = \beta_1 + \beta_1 X_i\ u_1$

Y_1 are normal Σ distributer

$f(Y_1 Y_2 \ldots\ldots Y_n / \beta_1 + \beta_2 X_i + \sigma^2)$

$= f(Y_1 / \beta_1 + \beta_2 X_i + \sigma^2)\ f(Y_2 / \beta_1 + \beta_2 X_i + \sigma^2) \ldots\ldots f(Y_n / \beta_1 + \beta_2) \quad \rightarrow (1)$

When $f(Y_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2} \frac{(Y_i - \beta_1 - \beta_1 X_i)^2}{\sigma^2}\right\} \quad \rightarrow (2)$

Exp mean e to the paru of expression indicator by { }

$f(Y_1) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\frac{(Y_i-\beta_1+\beta_2 X_i)^2}{\sigma^2}}$

Subtract (2) in (1)

$$f(Y_1, Y_2, Y_n / \beta_1 + \beta_2 X_1 + \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{Y_i-\beta_1-\beta_1 X_i}{\sigma^2}\right)^2\right\} \quad \rightarrow (3)$$

Y_1, Y_2, Y_n are known

But β_1 β_2 & σ_2 are not.

f so (3) is known as likelihood function.

Divided by LF (β_1,β_2, σ_2)

$$\therefore \text{LF}\ (\beta_1,\beta_2, \sigma_2) = \frac{1}{\sigma\sqrt{2\pi}}\ e\left\{-\frac{1}{2}\ \Sigma\ \left(\frac{Y_i-\beta_1-\beta_1 X_i}{\sigma^2}\right)^2\right\}$$

ML consists in estimating the unknown parameter in such a manner that the probability of observe give by Y's is highest as possible.

$$\text{In LF} = -n\ \ln\sigma - \frac{n}{2}\ln(2\pi) - \frac{1}{2}\Sigma\frac{(Y_1-\beta_1+\beta_2 X_i)^2}{\sigma^2} \quad \rightarrow (5)$$

Differencing (5) parameters with β_1,β_2 & σ_2

$$\frac{\partial mLF}{\partial \beta_1} = \frac{1}{\sigma^2}\ \Sigma\ (Y_1 - \beta_1 - \beta_2\ X_i)\ (-1) \quad \rightarrow (6)$$

$$\frac{\partial LnLF}{\partial \beta_2} = \frac{1}{\sigma^2}\ \Sigma\ (Y_1 - \beta_1 - \beta_2\ X_i)\ (-X_1) \quad \rightarrow (7)$$

$$\frac{\partial LnLF}{\partial \sigma_2} = -\frac{n}{\sigma} + \frac{1}{2\sigma^4}\Sigma(Y_1 - \beta_1 - \beta_2\ X_i)^2 \quad \rightarrow (8)$$

4.6 SUMMARY AND CONCLUSION:

This four-part course provides students with a basic foundation in econometrics combining theoretical underpinnings with practical problems. The first part is a review of the basic statistical concepts and matrix algebra. The second part introduces regression analysis: the basic idea behind the classical linear regression model (CLRM), the underlying assumptions, and the problem of estimation. Building on the two-variable model, it analyses a few extensions, the multiple regression model, and the matrix approach to the linear regression model. The third part of the course reviews hypothesis testing and interval estimation, both on the two-variable and multivariate regression models. The last part of the course analyzes the consequences on the estimators from relaxing the assumptions of the classical linear regression model, and discusses various remedies. It examines the cases of heteroskedasticity, autocorrelation, multicollinearity, non-linearity and non-stationary.

4.7 Lets sum it up:

In the concluding remarks, we can say that under the assumptions of the classical linear regression model (CLRM), we were able to show that the estimators of these parameters, $\hat{}\ \beta 1$, $\hat{}\ \beta 2$, and $\hat{}\sigma\ 2$, satisfy several desirable statistical properties, such as unbiasedness, minimum variance, etc. (Recall the BLUE property.) And after this by relaxing the assumptions of the classical linear regression model, we analyzed the consequences on the estimators.

4.8 EXCERCISES:

Q.1 Consider the following formulations of the two-variable PRF:

Model I: $Yi = \beta 1 + \beta 2Xi + ui$

Model II: $Yi = \alpha 1 + \alpha 2(Xi - .X) + ui$

a. Find the estimators of $\beta 1$ and $\alpha 1$. Are they identical? Are their variances identical?

b. Find the estimators of $\beta 2$ and $\alpha 2$. Are they identical? Are their variances identical?

c. What is the advantage, if any, of model II over model I?

Q.2 Let $r1$ = coefficient of correlation between n pairs of values (Yi , Xi) and $r2$ = coefficient of correlation between n pairs of values ($aXi + b$, $cYi + d$), where a, b, c, and d are constants. Show that $r1 = r2$ and hence establish the principle that the coefficient of correlation is invariant with respect to the change of scale and the change of origin.

Q.3 In the regression $Yi = \beta 1 + \beta 2Xi + ui$ suppose we multiply each X value by a constant, say, 2. Will it change the residuals and fitted values of Y? Explain. What if we *add* a constant value, say, 2, to each X value?

Q.4 Explain with reason whether the following statements are true, false, or uncertain:

a. Since the correlation between two variables, Y and X, can range from −1 to +1, this also means that cov (Y, X) also lies between these limits.

b. If the correlation between two variables is zero, it means that there is no relationship between the two variables whatsoever.

c. If you regress Yi on $\hat{}Yi$ (i.e., actual Y on estimated Y), the intercept and slope values will be 0 and 1, respectively.

Q.5 Regression without any regressor. Suppose you are given the model: $Yi = \beta 1 + ui$. Use OLS to find the estimator of $\beta 1$. What is its variance and the RSS? Does the estimated $\beta 1$ make intuitive sense? Now consider the two-variable model $Yi = \beta 1 + \beta 2Xi + ui$. Is it worth adding Xi to the model? If not, why bother with regression analysis?

4.9 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

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MODEL SPECIFICATION

1.1 INTRODUCTION:

In regression analysis specification is the process of developing a regression model. This process consists of selecting an appropriate functional form for the model and choosing which variables to include. As a first step of regression analysis, a person specifies the model. If an estimated model is misspecified, it will be biased and inconsistent.

Specification error occurs when an independent variable is correlated with the error term. There are several different causes of specification error:

- incorrect functional form
- a variable omitted from the model may have a relationship with both the dependent variable and one or more of the independent variables (omitted-variable bias);[2]
- an irrelevant variable may be included in the model
- the dependent variable may be part of a system of simultaneous equations (simultaneity bias)measurement errors may affect the independent variables

One of the assumptions of the classical linear regression model (CLRM) Assumption 9, is that the regression model used in the analysis is "Correctly" specified: If the model is not "Correctly" specified, we encounter the problem of model specification error or model specification bias. In this lesson we take a close and critical look at this assumption, because searching for the correct model is like searching for the Holy Grail.

1.2 OBJECTIVES:

1. Understand the model selection criteria for empirical analysis.

2. Understand the specification errors.

3 Understand the consequences of model specification errors on OLS estimates.

4. Detect specification errors through formal econometric tests.

5. Distinguish among the wide range of available tests for detecting specification errors.

1.5 MODEL SELECTION CRITERIA:

According to Hendry and Richard, model chosen for empirical analysis should satisfy the following criteria;

1. Be data admissible: that is, predictions made from the model must be logically possible.
2. Be consistent with theory; that is, it must make good economic sense. For example, if Milton Friedman's permanent income hypothesis holds, the intercept value in the regression of permanent consumption on permanent income is expected to be zero.
3. Have weakly exogenous regressors; that is, the explanatory variables, or regressors, must be uncorrelated with the error term.
4. Exhibit parameter constancy: that is, the value s of the parameters should be stable. Otherwise, forecasting will be difficulty. As Friedman notes, "The only relevant test of the validity of a hypothesis (Model) is comparison of its predictions with experience." In the absence of parameter constancy, such predictions will not be reliable.
5. Pure Random: Exhibit data coherency; that is, the residual estimated from the model must be purely random (technically, white noise). In other words, if the regression model is adequate, the residuals from this model must be white noise. If that is not the case, there is some specification error in the model. Shortly, we will explore the nature of specification error(s).
6. Be encompassing: that is the model should encompass or include all the rival models in the sense that it is capable of explaining their results. In short, other models cannot be an improvement over the chosen model.

1.6 TYPES OF SPECIFICATION ERRORS

Assume that on the basis of the criteria just listed we arrive at the model that we accept as a good model. To be concrete, let this model be

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + \beta_4 X_i^3 + u_{2i} \qquad \rightarrow (1)$$

Where Y = total cost of production and X=output. Equation (1) is the familiar text book example of the cubic total cost function,.

But suppose for some reason (say, laziness in plotting the scatter gram) a researcher decides to use the following model:

$$Y_i = \alpha_1 + \alpha_2 X_i + \alpha_3 X_i^2 + u_{2i} \qquad \rightarrow (2)$$

Note that we have changed the notation to distinguish this model from the true model.

Since (1) is assumed true, adopting (2) would constitute a specification error, the error consisting in omitting a relevant variable(X_i^3). Therefore, the error term u_{2i}. In (2) is in fact

$$u_{2i} = u_{1i} + \beta_4 X_i^3 \qquad \rightarrow (3)$$

We shall see shortly the importance of this relationship.

Now suppose that another researcher uses the following model;

$$Y_i = \lambda_1 + \lambda_2 X_i + \lambda_3 X_i^2 + \lambda_4 X_i^3 + \lambda_5 X_i^4 + u_{3i} \qquad \rightarrow (4)$$

If (1) is the "Truth," (4) also constitutes a specification error, the error here consisting in including an unnecessary or irrelevant variable in the sense that the true model assumes λ_5To be zero. The new error term is in fact

$$u_{3i} = u_{1i} - \lambda_5 X_i^4 \qquad \rightarrow (5)$$

$= u_{1i}$ Since $\lambda_5 = 0$ in the true model.

Now assume that yet another researcher postulates the following mode:

$$\text{In } Y_i = \gamma_1 + \gamma_2 X_i + \gamma_3 X_i^2 + \gamma_4 X_i^3 + u_{4i} \qquad \rightarrow (6)$$

In relation to the true model, (6) would also constitute a specification bias, the bias here being the use of the wrong functional form: In (1) Y appears linearly, whereas in (6) it appears log-olinearly.

Finally, consider the researcher who uses the following model:

$$Y_i^* = \beta_1^* + \beta_2^* X_i^* + \beta_3^* X_i^{*2} + \beta_4^* X_i^{*3} + u_i^* \qquad \rightarrow (7)$$

Where $Y_i^* = Y_i + \varepsilon_i$ and $X_i^* = X_i + w_i$, ε_i and w_i Being the errors of measurement. What (7) states is that instead of using the true Y_i And X_i we use their proxies, Y_i^* and X_i^* Which may contain errors of measurement. Therefore, in (7) we commit the errors of measurement bias. In applied work data are plagued by errors of approximations or errors of incomplete coverage or simply errors of omitting some observations. In the social sciences we often depend on secondary data and usually have no way of knowing the types of errors, if any, made by the primary data-collecting agency.

Another type of specification error relates to the way the stochastic error μ_i (or μ_t) enters regression model. Consider for instance, the following bivariate regression model without the intercept term;

$$Y_i = \beta X_i u_I \qquad \rightarrow (8)$$

Where the stochastic error term enters multiplicatively with the property that. satisfies the assumptions of the CLRM, against the following model

$$Y_i = \alpha X_i + u_i \qquad \rightarrow (9)$$

Where the error term enters additively. Although the variables are the same in the two models, we have denoted the slope coefficient in (8) by β and the sple coefificient in (9) by α Now if (8) is the "correct" or "true" model, would the estimated α provide an unbiased estimate of the true β^2 That is, will $E(\hat{\alpha}) = \beta$ If that is not the case, improper stochastic specification of the error term will constitute another source of specification error.

To sum up, in developing an impirical model, one is likely to commit one or more of the following specification errors:

1. Omission of a relevant variable(s)
2. Inclusion of an unnecessary variable(s)

3. Adopting the wrong functional form
4. Errors of measurement
5. In correct specification of the stochastic error term

Before turning to an examination of these specification errors in some detail, it may be fruitful to distinguish between model specification errors and model mis-specification errors. The first four types of error discussed above are essentially in the nature of model specification errors in that we have in mind a 'true" model but somehow we donot estimate the correct model. In model mis-specification errors, we do not know what the true model is to begin with. In this context one may recall the controversy between the Keynesians and the monetarists. The monetarists give primacy to money in explaining changes in GDP, whereas the Keynesians emphasize the role of government expenditure to explain changes in GDP. So to speak there are two competing models.

In what follows, we will first consider model specification errors and then examine model mis-specification errors.

1.5 CONSEQUENCES OF MODEL SPECIFICATION ERRORS

Whatever the sources of specification errors, what are the consequences? To keep the discussion simple, we will answer this question in the context of the three-variable model and consider in this section the first two types of specification errors discussed earlier, namely (1) underfitting a model, that is, omitting relevant variables, and (2) overfitting a model, that is, including unnecessary variables. Our discussion here can be easily generalized to more than two regressors, but with tedious algebra.., matric algebra becomes almost a necessity once we go beyond the three variable case.

1.5.1 Underfitting a Model (Omitting a Relevant Variable)

Suppose the true model is

$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

But for some reason we fit the following model:

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + v_i$$

The consequences of omitting variable X_3 are as follows:

1. If the left-out, or omitted, variable X_3 is correlated with the included variable X_2 that is r_{23}, the correlation coefficient between the two variables is nonzero, $\hat{\alpha}_1 and \hat{\alpha}_2$ are basied as well as inconsistent. That is $E(\hat{\alpha}_1) \neq \beta_1$ and $E(\hat{\alpha}_2) \neq \beta_2$ the bias does not disappear as the sample size get larger.
2. Even if X_2 and X_3 are not correlated $\hat{\alpha}_1$ although $\hat{\alpha}_2$ is now unbiased.
3. The disturbance variance σ^2 is incorrectly estimated.
4. The conventionally measured variance $\hat{\alpha}_1 (= \sigma^2 / \sum x_{2i}^2$ is a biased estimator of the variance of the true estimator $\hat{\beta}_1$
5. In consequence, the usual confidence interval and hypothesis-testing procedures are likely to give misleading conclusions about the statistical significance of the estimated parameters.
6. As another consequence, the forecasts based on the incorrect mode l and the forecast (confidence) intervals will be unreliable.

$$E(\hat{\alpha}_2) = \beta_2 + \beta_3 b_{32}$$

Where b_{32} is the slope in the regression of the excluded variable X_3 on the included variable X_2 ($b32 = \Sigma x_{3i} x_{2i} / \Sigma x_{2i}^2$). As shows, $\hat{\alpha}_2$ is biased, unless β_3 and β_{32} or both are zero. We rule out β_3 being zero, because in that case we do not have specification error to being with. The coefficient β_{32} will be zero if X_2 and X_3 are uncorrelated, which is unlikely in most economic data.

Now let us examine the variances of $\hat{\alpha}_2$ and $\hat{\beta}_2$

Var($\hat{\alpha}_2$) = $\frac{\sigma^2}{\Sigma x_{2i}^2}$

var($\hat{\beta}_2$) = $\frac{\sigma^2}{\Sigma x_{2i}^2(1-r_{23}^2)} = \frac{\sigma^2}{\Sigma x_{2i}^2}$ VIF

Where VIF (a measure of collinearity) is the variance inflation factor [=1/(1-r_{23}^2)] is the correlation coefficient between variable X_2 and X_3.

1.5.2 INCLUSION OF AN IRRELEVANT VARIABLE (OVERFITTING A MODEL)

Now let us assume that

$$Y_i = \beta_1 + \beta_2 X_{2i} + u_i$$

Is the truth, but we fit the following model.

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_i$$

And thus commit the specification error of including an unnecessary variable in the model.

The consequences of this specification error are as follows:

1. The OLS estimators of the parameters of the "incorrect" model are all unbiased and consistent, that is E(α_1) = β_1, $E(\hat{\alpha}_2)$= β_2,and $E(\hat{\alpha}_3)$= β_3 =0
2. 2. The error variance σ^2 Is correctly estimated.
3. The usual confidence interval and hypothesis-testing procedures remain valid.
4. However, the estimated α's will be generally inefficient, that is, their variances will be generally larger than those of the $\hat{\beta}$s of the true model.

From the usual OLS formula we know that

Var ($\hat{\beta}_2$) =$\frac{\sigma^2}{\Sigma x_{2i}^2}$

and Var($\hat{\alpha}_2$) =$\frac{\sigma^2}{\Sigma x_{2i}^2(1-r_{23}^2)}$

Therefore $\frac{\text{Var}(\hat{\alpha}_2)}{\text{var}(\hat{\beta}_2)} = \frac{1}{1-r_{23}^2}$

Since $0 \leq r_{23}^2 \leq 1$, it follows that $(\hat{\alpha}_2) \geq \text{var}(\hat{\beta}_2)$; that is, the variance of $\hat{\alpha}_2$ is generally greater than the variance of $\hat{\beta}_2$ even though, on average $\hat{\alpha}_2 = \hat{\beta}_2$.

The implication of this finding is that the including of the unnecessary variable X_3 makes the variance of $\hat{\alpha}_2$ larger than necessary, thereby making $\hat{\alpha}_2$ less precise. This is also true of $\hat{\alpha}_1$

1.6 TESTS OF SPECIFICATION ERRORS

1.6.1 Detecting the presence of unnecessary variables (Over fitting a model)

Suppose we develop a K-variable model to explain a phenomenon:

$Y_i = \beta_1 + \beta_2 X_{2i} + \ldots\ldots\ldots\ldots + \beta_k X_{ki} + u_i$

However, we are not totally sure that, say, the variable X_k really belongs in the model. One simple way to find this out is to test the significance that we are not sure whether, say β_k with the usual *t* test: $_t = \hat{\beta}_k / se(\hat{\beta}_k)$ But suppose that we are not sure whether, say, X_3 and X_4 legitimately belong in the model. This can be easily ascertained by the F test. Thus, detecting the presence of an irrelevant variable(or variables) is not a difficult task.

It is, however, very important to remember that in carrying out these tests of significance we have a specific model in mind. We accept that model as the maintained

hypothesis or the "truth," however tentative it may be. Given that model, then, we can find out whether one or more regressors are really relevant by the usual *t* and *f* tests. But note carefully that we should not use the *t* and *f* tests to build a model iteratively, that is, we should not say that initially Y is related to X_2 only because $\hat{\beta}_2$ is statistically significant and then expand the model to include X_3 and decide to keep that variable in the model if $\hat{\beta}_3$ turns out to be statistically significant, and so on. This strategy of building model is called the bottom-up approach (starting with a smaller model and expanding it as one goes along) or by the somewhat pejorative term, data mining (other names are regression fishing, data grubbing, data snooping , and number crunching).

1.6.2 Tests for Omitted Variables and incorrect functional form

In practice we are never sure that the model adopted for empirical testing is "the truth, the whole truth and nothing but the truth." On the basis of theory or introspection and prior empirical work, we develop a model that we believe captures the essence of the subject under study. We then subject the model to empirical testing. After we obtain the results, we being the post mortem, keeping in mind the criteria of a good model discussed earlier. It is at this stage that we come to know if the chosen model is adequate. In determining model adequacy, we look at some broad features of the results, such as the R^2 value, the estimated coefficients in relation to their prior expectations, the Durbin-Watson statistic, and the like. If these diagnostics are reasonably good, we proclaim that the chosen model is a fair representation of reality. By the same token, if the results do not look encouraging because the R^2 value is too low or because very few coefficients are statistically significant or have the correct signs or because the Durbin-Watson *d* is too low, then we being to worry about model adequacy and look for remedies. May we have omitted an important variable, or have

used the wrong functional form, or have not first differenced the time series (to remove serial correlation), and so on.

1.6.3 The Durbin-Watson *d* Statistics Once Again.

If we examine the routinely calculated Durbin-Watson d we see that for the linerar cost function the estimated *d* suggesting that there is positive "correlation" in the estimated residuals: for n = 10 and k' = 1 and then 5 percent *d* critical value are d_L Liewise, the computed value for the quadratic cost function is 1..38, whereas the 5 percent critical values are d_L = 0.697 and D_U= 1.641, indicating indecision. But if we use the modified *d* test we can say that there is positive "correlation" in the residuals, for the computed d is less than d_U. For the cubic cost function, the true specification, the estimated *d* value does not indicate any positive "correlation" in the residuals.

The observed positive "correlation" in the residuals when we fit the linear or quadratic model is not a measure of (first oder) serial correlation but of fact that some variable(s) that belong in the modeol are included in the error term and need to be culled out from it and introduced in their own right as explanatory variables: If we exclude the $x_1^3$from the cost function, the error term in the mis-specified model is in fact ($\mu_{li} + \beta_4 X_1^3$and it will exhibit a systematic pattern (e.g. positive autocorrelation) if X_1^3in fact affects Y significantly.

To use the Durbin-Watson test for detecting model specification error(s), we proceed as follows

1. From the assumed mode, obtain the OLS residuals.
2. If it is believed that the assumed model is mis-specified because it excludes a relevant explanatory variable, say, Z from the model, order the residuals obtained in Step 1 according to increasing values of Z. Note: The Z variable

could be one of the x variables included in the assumed model or it could be some function of that variable, such as X^2 and X^3.

3. Compute the d statistic from the residuals thus ordered by the usual d formula, namely

$$d = \left(\frac{\sum_{t=2}^{n} (\hat{\mu}_t - \hat{\mu}_{t-1})^2}{\sum_{t=1}^{n} \hat{\mu}_t^2} \right)$$

Note: The subscript t is the index of observation here and does not necessarily mean that the date are time series.

4. From the Durbin-Watson tables, if the estimated d value is significant, then one can accept the hypothesis of model mis-specification. If that turns out to be the case, the remedial measures will naturally suggest themselves. Ramsey's Reset Test. Ramsey has porposed a general test of specification error called RESET (regression specification error test0. Here we will illustrate only the simplest version of the test. To fix ideas, let us continue with out cost-output example that the cost function is linerar in output as.

   $Y_i = \lambda_1 + \lambda_1 X_i + \mu_{3i}$

Where Y= total cost and X= output. Now if we plot the residuals $\hat{\mu}_i$ obtained from this regression against $\hat{Y}_i$ the estimated Y_i from this model, we get the picture shown in figure Although $\sum \hat{\mu}_i$ and $\sum \hat{\mu}_i \hat{Y}_i$ are necessarily zero.

the residuals in this figure show a pattern in which theirmean changes systematically with $\hat{Y}_i$. This would suggest that if we introduce $\hat{Y}_i$ in some form as regressor (s), it should increase R^2. And if the increase in, R^2 is statistically significant (on the basis of the F test discussed in previous Lesson), it would suggest that the liner cost function was mis-specified. This sis essentially the idea behind RESET. The steps involved in RESET are as follow:

1. From the chosen model, obtain the estimated Y_i, that is $\hat{Y}_1$.

   $Y_i = \lambda_1 + \lambda_2 X_i + u_{3i}$

2. Rerun (13.4.6) introducing $\hat{Y}_1$ in some form as an additional regressor(s). From Figure ,we observe that there is a curvilinear relationship between $\hat{\mu}_i$ and $\hat{Y}_i$. Suggesting that one can introduce $\hat{Y}_i^2 and \hat{Y}_i^3$ as additional regressors Thus, we run.

   $$Y_i = \beta_1 + \beta_2 X_i + \beta_3 \hat{Y}_i^2 + \beta_4 \hat{Y}_i^3 + u_i$$

3. Let the R^2 obtained from be R^2_{new} and that obtained from be R^2_{old} Then we can use the F test first introduced in namely.

   $$F = \frac{(R^2_{new} - R^2_{old})/\text{number of regressors}}{(1 - R^2_{new})/(\text{n} - \text{number of parametres in the new model}}$$

   to find out if the increase in R^2 from using is statistically significant.

4. **Lagrange Multiplier (LM) Test for Adding Variables**. This is an alternative to Ramsey's RESET test. To illustrate this test, we will continue with the preceding illustrative example.

If we compare the linear cost function with the cubic cost function the former is a restricted version of the latter.The restricted regression assumes that the coefficients of the squared and cubed output terms are equal to zero. To test this, the LM test proceeds as follows;

1. Estimate the restricted regression by OLS and obtain the residuals $\hat{u}_i$.

2. If in fact the unrestricted regression is the true regression the residuals obtained in should be related to the squared and cubed output terms, that is $X_i^2$and X_i^3
3. This suggests that we regress the $\hat{u}_i$obtained in Step 1 on all the regressors(including those in the restricted regression) which in the present case means.

   $\hat{u}_i = \alpha_1 + \alpha_2 X_i + \alpha_3 X_i^2 + \alpha_4 X_i^3 + \nu_i$

Where v is an error term with the usual properties.

4. For large-sample size, Engle has shown that n(the sample size0 times the R^2 Estimated from the (auxiliary) regression follows the chisquare distribution with df equal to the number of resrtrictions imposed by the restricted regression, two in the present example since the terms X_i^2 and $X_i^3$are dropped from the model. Symbolically, we write.

$nR^2 \underset{asy}{\sim} X^2_{\text{(number of restrictions)}}$

   Where as Y means asymptotically, that is, in large samples.

5. If the chi-square value obtained from exceeds the critical chi-square value at the chosen level of significant, we reject the restricted regression. Otherwise, we do not reject it.

1.7 SUMMARY AND CONCLUSIONS:

1. The assumption of the CLRM that the econometric model used in analysis is correctly specified has two meanings. One, there are no equation specification errors, and two, there are no model specification errors. In this lesson the major focus was on equation specification errors.

2. The equation specification errors discussed in this lesson were

(1) omission of important variable(s), (2) inclusion of superfluous variable(s), (3) adoption of the wrong function form, (4) incorrect specification of the error term ui, and (5) errors of measurement in the regressand and regressors.

3. When legitimate variables are omitted from a model, the consequences can be very serious: The OLS estimators of the variables retained in the model not only are biased but are inconsistent as well. Additionally, the variances and standard errors of these coefficients are incorrectly estimated, thereby vitiating the usual hypothesis-testing procedures.

4. The consequences of including irrelevant variables in the model are fortunately less serious: The estimators of the coefficients of the relevant as well as "irrelevant" variables remain unbiased as well as consistent, and the error variance σ2 remains correctly estimated. The only problem is that the estimated variances tend to be larger than necessary, thereby making for less precise estimation of the parameters. That is, the confidence intervals tend to be larger than necessary.

5. To detect equation specification errors, we considered several tests, such as (1) examination of residuals, (2) the Durbin–Watson *d* statistic, (3) Ramsey's RESET test, and (4) the Lagrange multiplier test**.**

6. A special kind of specification error is errors of measurement in the values of the regressand and regressors. If there are errors of measurement in the regressand only, the OLS estimators are unbiased as well as consistent but they are less efficient. If there are errors of measurement in the regressors, the OLS estimators are biased as well as inconsistent.

7. Even if errors of measurement are detected or suspected, the remedies are often not easy. The use of instrumental or proxy variables is theoretically attractive but not always practical. Thus it is very important in practice that the researcher be careful in stating the sources of his/her data, how they were collected, what definitions were used, etc. Data collected by

official agencies often come with several footnotes and the researcher should bring those to the attention of the reader

1.8 LETS SUM IT UP:

In last, we can say that specification error occurs when an independent variable is correlated with the error term. In this process we find appropriate functional form for the model and choosing which variables to include. If particular estimated model is mis-specified, it will give biased and inconsistent results.

1.9 EXCERCISES :

Q.1 Consider the model

$$Yi = \beta1 + \beta2X^*I + ui$$

In practice we measure X*Xi such that

a. $Xi = X^*i + 5$

b. $Xi = 3X^*i$

c. $Xi = (X^*i + \varepsilon i)$, where εi is a purely random term with the usual properties

What will be the effect of these measurement errors on estimates of true

$\beta1$ and $\beta2$?

Q.2 Suppose that the true model is

$$Yi = \beta1Xi + ui \qquad (1)$$

but instead of fitting this regression through the origin you routinely fit the usual intercept-present model:

$$Yi = \alpha 0 + \alpha 1Xi + vi \qquad (2)$$

Assess the consequences of this specification error

Q.3 Suppose that the "true" model is

$$Yi = \beta 1 + \beta 2X2i + ut \qquad (1)$$

but we add an "irrelevant" variable X3 to the model (irrelevant in the sense that the true $\beta 3$ coefficient attached to the variable X3 is zero) and

estimate

$$Yi = \beta 1 + \beta 2X2i + \beta 3X3i + vi \qquad (2)$$

a. Would the R2 and the adjusted R2 for model (2) be larger than that for model (1)?

b. Are the estimates of $\beta 1$ and $\beta 2$ obtained from (2) unbiased?

c. Does the inclusion of the "irrelevant" variable X3 affect the variances of $\hat{}\ \beta 1$ and $\hat{}\ \beta 2$?

Q.4 what are the consequences of model specification errors?

Q.5 What are the various tests used for detecting specification errors?

1.10 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

NESTED VERSUS NON - NESTED MODELS

2.1 INTRODUCTION:

In carrying out specification testing, it is useful to distinguish between nested and non-nested mode,s. To distinguished between the two, consider the following models:

Model A: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + u_i$

Model B: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

We say that Model B is nested in Model A because it is a special case of Model A: if we estimate Model A and test the hyp0othesis that $\beta_4 = \beta_5 = 0$ and do not reject it on the basis of , say, the F test Model A reduces to Model B. If we add variable X_4 to Model B, then Model A will reduce to Model B if β_5 is zero; here we will use the t test tot est the hypothesis that the coefficient of X_5 is zero.

Without calling them such, the specification error tests we have discussed previously and the restricted F are essentially tests of nested hypothesis.

Now consider the following modes:

Model C: $Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_i$

Model D: $Y_i = \beta_1 + \beta_2 Z_{2i} + \beta_3 Z_{3i} + u_i$

Where the X's And Z's are different variables. We say that Models C and D are non-nested because one cannot be derived as a special case of the other. In economics, as in other sciences, more than one competing theory may explain a phenomenon. Thus the monetarists would emphasize the role of money in explaining changes in GDP, whereas the Keynesians may explain them by changes in government expenditure.

It may be noted here that one can allow Model C and D to contain regressors that are common to both. For example, X_3 could be included in Model D and Z_2 could be included in Model C. Even then these are non nested models, because Model C does not contain Z_3 and Model D does not contain X_2.

Even if the same variables enter the model, the functional form may make two models non-nested. For example, consider the model:

Model E: $Y_i = \beta_1 + \beta_2 InZ_{2i} + \beta_3 \text{In}Z_{3i} + w_i$

Models D and E are non-nested, as one cannot be derived as a special case of the other.

Since we already have looked at tests of nested model (t and F tests), in the following section we discuss some of the tests of non-nested model, which earlier we called model mis-specification errors.

2.2 OBJECTIVES:

1. The first objective is to distinguish between nested and non-nested models.

2. Understand the model selection criteria for empirical analysis.

3. Detect nested and non-nested models through formal econometric tests.

4. Distinguish among the wide range of available tests for detecting non-nested models.

2.3 TESTS OF NON-NESTED HYPOTHESES

According to Harvey, there are two approaches to testing non-nested hypotheses:

(1) the **discrimination approach,** where given two or more competing models, one chooses a model based on some criteria of goodness of fit, and (2) the **discerning approach** (my terminology) where, in investigating one model, we take into account information provided by other models. We consider these approaches briefly.

2.3.1 The Discrimination Approach:

Consider Models C and D above. Since both models involve the same dependent variable, we can choose between two (or more) models based on some goodness-of-fit criterion, such as $R2$ or adjusted $R2$, which we have already discussed. But keep in mind that in comparing two or more models, the regress and must be the same. Besides these criteria, there are other criteria that are also used. These include **Akaike's information criterion (AIC), Schwarz's information criterion (SIC),** and **Mallows's *Cp* criterion.**

2.3.2 The Discerning Approach:

The Non-Nested *F* Test or Encompassing *F* Test. Consider Models C and D introduced earlier. How do we choose between the two models? For this purpose suppose we estimate the following nested or *hybrid* model:

Model F: $Yi = \lambda 1 + \lambda 2X2i + \lambda 3X3i + \lambda 4Z2i + \lambda 5Z3i + ui$

Notice that Model F *nests or encompasses* models C and D. But note that C is not nested in D and D is not nested in C, so they are non-nested models.

Now if Model C is correct, $\lambda 4 = \lambda 5 = 0$, whereas Model D is correct if $\lambda 2 = \lambda 3 = 0$. This testing can be done by the usual F test, hence the name non-nested F test.

However, there are problems with this testing procedure. *First,* if the *X*'s and the *Z*'s are highly correlated, then, as noted in the lesson on multicollinearity, it is quite likely that one or more of the λ's are individually statistically insignificant, although on the basis of the *F* test one can reject the hypothesis that all the slope coefficients are simultaneously zero. In this case, we have no way of deciding whether Model C or Model D is the correctmodel. *Second,* there is another problem. Suppose we choose Model C as the *reference hypothesis* or model, and find that all its coefficients are significant. Now we add *Z*2 or *Z*3 or both to the model and find, using the *F* test, that their incremental contribution to the explained sum of squares (ESS) is statistically insignificant. Therefore, we decide to choose Model C. But suppose we had instead chosen Model D as the reference model and found that all its coefficients were statistically significant. But when we add *X*2 or *X*3 or both to this model, we find, again using the *F* test, that their incremental contribution to ESS is insignificant. Therefore, we would have chosen model D as the correct model. Hence, "the choice of the reference hypothesis could determine the outcome of the choice model,"33 especially if severe multicollinearity is present in the competing regressors. *Finally,* the artificially nested model *F* may not have any economic meaning.

2.3.3 Davidson–MacKinnon *J* Test.

Because of the problems just listed in the non-nested *F* testing procedure, alternatives have been suggested. One is the *Davidson–MacKinnon J* test. To illustrate this test, suppose we want to compare hypothesis or Model C with hypothesis or Model D. The ***J* test** proceeds as follows:

1. We estimate Model D and from it we obtain the estimated *Y* values, $\hat{Y}Di$.

2. We add the predicted *Y* value in Step 1 as an additional regressor to

Model C and estimate the following model:

$$Yi = \alpha 1 + \alpha 2X2i + \alpha 3X3i + \alpha 4\hat{}YDi + ui \qquad \textbf{(5)}$$

where the $\hat{}YDi$ values are obtained from Step 1. This model is an example of the **encompassing principle,** as in the Hendry methodology.

3. Using the *t* test, test the hypothesis that $\alpha 4 = 0$.

4. If the hypothesis that $\alpha 4 = 0$ is not rejected, we can accept (i.e., not

reject) Model C as the true model because $\hat{}Y\,Di$ included in (5), which represent the influence of variables not included in Model C, have no additional explanatory power beyond that contributed by Model C. In other words, Model C *encompasses* Model D in the sense that the latter model does not contain any additional information that will improve the performance of Model C. By the same token, if the null hypothesis is rejected, Model C cannot be the true model (why?).

5. Now we reverse the roles of hypotheses, or Models C and D. We now estimate Model C first, use the estimated *Y* values from this model as regressor in (5), repeat Step 4, and decide whether to accept Model D over Model C. More specifically, we estimate the following model:

$$Yi = \beta 1 + \beta 2Z2i + \beta 3Z3i + \beta 4\hat{}YCi + ui \qquad \textbf{(6)}$$

where $\hat{}Y\,Ci$ are the estimated *Y* values from Model C. We now test the hypothesis that $\beta 4 = 0$. If this hypothesis is not rejected, we choose Model D over C. If the hypothesis that $\beta 4 = 0$ is rejected, choose C over D, as the latter does not improve over the performance of C.

Although it is intuitively appealing, the *J* test has some problems. Since the tests given in (5) and (6) are performed independently, we have the following likely outcomes

Hypothesis: $\alpha 4 = 0$

Hypothesis: $\beta 4 = 0$	**Do not reject**	**Reject**
Do not reject	Accept both C and D	Accept D, rejectC
Reject	Accept C, reject D	Reject both C and D

As this table shows, we will not be able to get a clear answer if the *J* testing procedure leads to the acceptance or rejection of both models. In case both models are rejected, neither model helps us to explain the behavior of *Y*. Similarly, if both models are accepted, as Kmenta notes, "the data are apparently not rich enough to discriminate between the two hypotheses

[models]." Another problem with the *J* test is that when we use the *t* statistic to test the significance of the estimated *Y* variable in models (5) and (6), the *t* statistic has the standard normal distribution only asymptotically, that is, in large samples. Therefore, the *J* test may not be very powerful (in the statistical sense) in small samples because it tends to reject the true hypothesis or model more frequently than it ought to.

2.4 SUMMARY AND CONCLUSIONS:

If errors of measurement are detected or suspected, the remedies

are often not easy. The use of instrumental or proxy variables is theoretically attractive but not always practical. Thus it is very important in practice that the researcher be careful in stating the sources of his/her data, how they were collected, what definitions were used, etc. Data collected by official agencies often come with several footnotes and the researcher should bring those to the attention of the reader. Model mis-specification errors can be as serious as equation specification errors. In particular, we distinguished between nested and nonnested models. To decide on the appropriate model we discussed the nonnested, or encompassing, *F* test and the Davidson–MacKinnon *J* test and pointed out the limitation of each test.

2.5 LETS SUM IT UP:

In concluding remarks, we can say that Model mis- specification errors can lead to various equation specification errors. In this lesson, we distinguished between nested and non-nested models. Hendry argues several econometric work starts with very simplified models and that not enough diagnostic tests are applied to check whether something is wrong with the maintained model. His suggested strategy is to start with a very general model and then progressively simplify it by some data based simplification tests.

2.6 EXCERCISES:

Q.1 Distinguish between nested and non-nested models?

Q.2 What is the discrimination approach of non nested hypotheses?

Q.3 Elaborate the discerning approach of non nested hypotheses?

Q.4 What is Davidson–MacKinnon J Test

2.7 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

4. Green, W.(2000). Econometrics, Prentice Hall of India, New Delhi.

UNIT - III

3.1 INTRODUCTION:

The assumption 10 of the classical linear regression model (CLRM) is that there is no multicollinearity among the regressors included in the regression model. In this lesson we take a critical look at this assumption by seeking answers to the following questions:

1. What is the nature of multicollinearity?

2. Is multicollinearity really a problem?

3. What are its practical consequences?

4. How does one detect it?

5. What remedial measures can be taken to alleviate the problem of multicollinearity?

3.2 OBJECTIVES:

1. Understand the meaning of multicollinearity.

2. Understand the consequences of multicollinearity on OLS estimates.

3. Detect multicollinearity. through rule of thumb inspection.

4. Detect multicollinearity. through formal econometric tests.

5. Distinguish among the wide range of available tests for detecting multicollinearity..

3.3 MULTICOLLINEARITY

It means the existence of a perfect or exact linear relationship among some all explanatory variables of a regression model.

$X_{2i} = \lambda X_{3i}$ $\rightarrow$ perfect multicollinearity

It is due to Ragnar Frisch

$\lambda_1 X_1 + \lambda_2 X_2 + \ldots\ldots\ldots\ldots + \lambda_k X_k = 0$

$\lambda_1\ \lambda_2 \ldots\ldots\ldots\ldots \lambda_k$ are constants

The term multicollinearity is used in a broader sense to include the case of perfect multicollionearity.

$\lambda_1 X_1 + \lambda_2 X_2 + \ldots\ldots\ldots\ldots + \lambda_k X_k + V_i = 0$

Where V_i is a stochastic error term.

Difference between perfect & less than perfect multicollinearly assumed.

$$X_{2i} = -\frac{\lambda_1}{\lambda_2} X_{1i} - \frac{\lambda_3}{\lambda_2} X_{3i} - \cdots - \frac{\lambda_k}{\lambda_2} X_{ki}$$

Linear combination $\lambda_2 \neq 0$

$$X_{2i} = -\frac{\lambda_1}{\lambda_2} X_{1i} - \frac{\lambda_3}{\lambda_2} X_{3i} - \cdots - \frac{\lambda_k}{\lambda_2} X_{ki} - \frac{1}{\lambda_2} v_i$$

which shows that X_2 is not an exact linear. Combination of other X's because it is also determined by the stochastic error term V_i.

3.3.1 NATURE/ SOURCE:

If multicollinearity is perfect in the sense, the regression coefficients of the *X* variables are indeterminate and their standard errors are infinite. If multicollinearity is less than perfect, the regression coefficients, although determinate, possess large standard errors (in relation to the coefficients themselves), which means the coefficients cannot be estimated with great precision or accuracy. The following can be reasons for the existence of multicollinearity:

1. Data collection method
2. Constraints on the model.
3. Model specification.
4. An over determined model.

3.3.2 REMEDIAL MEASURES

3.3.2.1 Do Nothing

The "do nothing" school of thought is expressed by Blanchard as follows:

When students run their first ordinary least squares (OLS) regression, the first problem that they usually encounter is that of multicollinearity. Many of them conclude that there is something wrong with OLS; some resort to new and often creative techniques to or around the problem. But we tell them, this is wrong, Multicollineaity is God's will, not a problem with OLS or statistical technique in general.

What Blanchard is saying is that multicollinearity is essentially a data deficiency problem (micronumerosity, again) and some times we have no choice over the data we have available for empirical analysis.

3.3.2.2 Rule of Thumb Procedures

One can try the following rules of thumb to ad

dress the problem of multicollinearity, the success depending on the severity of the multicollinearity problem.

1. **A priori information.** Suppose we consider the model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where Y = consumption, X_2 = income, and X_3 = wealth. As noted before, income and wealth variables tend to be highly collinear. But suppose a priori we believe that $\beta_3 = 0.10\beta_2$; that is, the rate of change of consumption with respect to wealth is one-tenth the corresponding rate with respect to income. We can then run the following regression:

$$\begin{aligned} Y_i &= \beta_1 + \beta_2 X_{2i} + 0.10\beta_2 X_{3i} + u_i \\ &= \beta_1 + \beta_2 X_i + u_i \end{aligned}$$

Where $X_i + 0.1X_{3i}$. Once we obtain $\hat{\beta}_2$, we can estimate $\hat{\beta}_3$ from the postulated relationship between β_2 and β_3.

2. **Combining cross-sectional and time series data.** A variant of the extraneous or a priori information technique is the combination of cross-sectional and time-series data, known as pooling the data. Suppose we want to study the demand for automobiles in the United States and assume we have time series data on the number of cars sold, average price of the car,

$$\ln Y_t = \beta_1 + \beta_2 \ln P_t + \beta_3 \ln I_t + u_t$$

Where Y = number of cars sold, P = average price, I = income, and t = time. Out objective is to estimate the price elasticity β_2 and income elasticity β_3.

In time series data the price and income variables generally tend to be highly collinear. Therefore, if we run the proceeding regression, we shall be faced with the usual multicollinearity problem. A way out of this has been suggested by Tobin. He says that if we have cross-sectional data (for example, data generated by consumer panels, or budget studies conducted by various private and governmental agencies), we can obtain a fairly reliable estimate of the income elasticity β_3 because in such data, which are at a point in time, the prices do not vary much. Let the cross-sectionally estimated income elasticity be $\hat{\beta}_3$. Using this estimate, we may write the preceding times series regression as

$$Y_t^* = \beta_1 + \beta_2 \ln P_t + u_t$$

Where Y* = In Y - $\hat{\beta}_3$ In I, that is, Y* represents that value of Y after removing from it the effect of income. We can now obtain an estimate of the price elasticity β_2 from the preceding regression.

3) Dropping a variable (s) and specification bias. When faced with severe multicollinearity, one of the "simplest" things to do is to drop one of the collinear variables. Thus, in our consumption-income-wealth illustration, which shows that, whereas in the original model the income variable was statistically insignificant, it is now 'highly' significant.

But in dropping a variable from the model we may be committing specification bias or specification error. Specification bias arises from incorrect specification of the model used in the analysis. Thus, if economic theory says that income and wealth should both

be included in the model explaining the consumption expenditure, dropping the wealth variable would constitute specification bias.

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

But we mistakenly fit the model

$$Y_i = b_1 + b_{12} X_{2i} + \hat{u}_i \quad \ldots\ldots\ldots\ldots (1)$$

Then it can be shown that

$$E(b_{12}) = \beta_2 + \beta_3 b_{32} \quad \ldots\ldots\ldots\ldots (2)$$

where b_{32} = slope coefficient in the regression of X_3 on X_2. Therefore, it is obvious that b_{12} will be a biased estimate of β_2 as long as b_{32} is different from zero (it is assumed that β_3 is different from zero; otherwise there is no sense in including X_3 in the original model). Of course, if b_{32} is zero, we have no multicollinearity problem to begin with. It is also clear from that if both b_{32} and β_3 are positive (or both are negative), $E(b_{12})$ will be greater than β_2; hence, on the average b_{12} will overestimate β_2, leading to a positive bias. Similarly, if the product $b_{32}\beta_3$ is negative, on the average b_{12} will underestimate β_2, leading to a negative bias.

4) Transformation of variables. Suppose we have time series data on consumption expenditure, income and wealth. One reason for high multicollinearity between income and wealth in such data is that over time both the variables tend to move in the same direction. One way of minimizing this dependence is to proceed as follows.

If the relation

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \quad \ldots\ldots\ldots\ldots (3)$$

Holds at time t, it must also hold at time t – 1 because the origin of time is arbitrary anyway. Therefore, we have

$$Y_{t-1} = \beta_1 + \beta_2 X_{2,t-1} + \beta_3 X_{3,t-1} + u_{t-1} \quad \ldots\ldots\ldots (4)$$

If we subtract (3) from (1), we obtain

$$Y_t - Y_{t-1} = \beta_2 (X_{2t} - X_{2,t-1}) + \beta_3 (X_{3t} - X_{3,t-1}) + v_t \quad \ldots\ldots (5)$$

Where $v_t = u_t - u_{t-1}$. Equation (5) is known as the first difference form because we run the regression, not on the original variables, but on the differences of successive values of the variables.

The first difference regression model often reduces the severity of multicollinearity because, although the levels of X_2 and X_3 may be highly correlated, there is no a priori reason to believe that their differences will also be highly correlated.

As we shall see in the lessons on time series econometrics, an incidental advantage of the first – difference transformation is that it may make a nonstationary time series stationary. In those lessons we will see the importance of stationary time series Another commonly used transformation in practice is the ratio transformation.

Consider the model:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \quad \ldots\ldots\ldots (6)$$

Where Y is consumption expenditure in real dollars, X_2 is GDP, and X_3 is total population. Since GDP and population grow over time, they are likely to be correlated. One “Solution” to this problem is to express the model on a per capita basis, that is, by dividing (6) by X_3, to obtain:

$$\frac{Y_t}{X_{3t}} = \beta_1\left(\frac{1}{X_{3t}}\right) + \beta_2\left(\frac{X_{2t}}{X_{3t}}\right) + \beta_3 + \left(\frac{u_t}{X_{3t}}\right) \quad \text{.....(7)}$$

Such a transformation may reduce collinearity in the original variables.

But the first – difference or ratio transformations are not without problems. For instance, the error term v_t in ( ) may not satisfy one of the assumptions of the classical linear regression model, namely, that the disturbances are serially uncorrelated.

5) Additional or new data. Since multicollinearity is a sample feature, it is possible that in another sample involving the same variables collinearity may be so serious as in the first sample. Sometimes simply increasing the size of the sample (if possible) may attenuate the collinearity problem. For example, in the three-variable model we saw that

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_{2i}^2 (1 - r_{23}^2)}$$

Now as the sample size increases, $\sum x_{2i}^2$ will generally increase. Therefore, for any given r_{23}, the variance of $\hat{\beta}_2$ will decrease, thus decreasing the standard error, which will enable us to estimate β_2 more precisely.

6) Other methods of remedying multicollinearity. Multivariate statistical techniques such as factor analysis and principal components or techniques such as ridge regression are often employed to 'solve' the problem of multicollinearity. Unfortunately, these techniques are beyond the scope of this book, for they cannot be discussed competently without resorting to matrix algebra.

3.4 SUMMARY AND CONCLUSIONS:

1. One of the assumptions of the classical linear regression model is that there is no multicollinearity among the explanatory variables, the X's. Broadly interpreted, multicollinearity refers to the situation where there is either an exact or approximately exact linear relationship among the X variables.

2. The consequences of multicollinearity are as follows: If there is perfect collinearity among the X's, their regression coefficients are indeterminate and their standard errors are not defined. If collinearity is high but not perfect, estimation of regression coefficients is possible but their standard errors tend to be large. As a result, the population values of the coefficients cannot be estimated precisely. However, if the objective is to estimate linear combinations of these coefficients, the estimable functions, this can be done even in the presence of perfect multicollinearity

3. Although there are no sure methods of detecting collinearity, there are several indicators of it, which are as follows:

(a) The clearest sign of multicollinearity is when R2 is very high but none of the regression coefficients is statistically significant on the basis of the conventional t test. This case is, of course, extreme.

(b) In models involving just two explanatory variables, a fairly good idea of collinearity can be obtained by examining the zero-order, or simple, correlation coefficient between the two variables. If this correlation is high, multicollinearity is generally the culprit.

(c) However, the zero-order correlation coefficients can be misleading in models involving more than two X variables since it is possible to have low zero-order correlations and yet find high multicollinearity. In situations like these, one may need to examine the partial correlation coefficients.

(d) If R2 is high but the partial correlations are low, multicollinearity is a possibility. Here one or more variables may be superfluous. But if R2 is high and the partial correlations are also

high, multicollinearity may not be readily detectable. Also, as pointed out by C. Robert, Krishna Kumar, John O'Hagan, and Brendan McCabe, there are some statistical problems with the partial correlation test suggested by Farrar and Glauber.

(e) Therefore, one may regress each of the Xi variables on the remaining X variables in the model and find out the corresponding coefficients of determination R2 . A high R2 would suggest that Xi is highly correlated with the rest of the X's. Thus, one may drop that Xi from the model, provided it does not lead to serious specification bias.

4. Detection of multicollinearity is half the battle. The other half is concerned with how to get rid of the problem. Again there are no sure methods, only a few rules of thumb. Some of these rules are as follows: (1) using extraneous or prior information, (2) combining cross-sectional and time series data, (3) omitting a highly collinear variable, (4) transforming data, and (5) obtaining additional or new data. Of course, which of these rules will work in practice will depend on the nature of the data and severity of the collinearity problem.

5. We noted the role of multicollinearity in prediction and pointed out that unless the collinearity structure continues in the future sample it is hazardous to use the estimated regression that has been plagued by multicollinearity for the purpose of forecasting.

3.5 LETS SUM IT UP:

In the concluding remarks, we can say that in cases of near or high multicollinearity, one is likely to encounter the following consequences:

1. Although BLUE, the OLS estimators have large variances and covariances, making precise estimation difficult.

2. Because of consequence 1, the confidence intervals tend to be much wider, leading to the acceptance of the "zero null hypothesis" (i.e., the true population coefficient is zero) more readily.

3. Also because of consequence 1, the *t* ratio of one or more coefficients tends to be statistically insignificant.

4. Although the *t* ratio of one or more coefficients is statistically insignificant, *R*2, the overall measure of goodness of fit, can be very high.

5. The OLS estimators and their standard errors can be sensitive to
small changes in the data.

3.6 EXCERCISES:

Q.1 What do you mean by multicollinearity?

Q.2 What is Rule of Thumb?

Q.3 How can we detect multicollinearity?

Q.4.State with reason whether the following statements are true, false, or uncertain:

a. Despite perfect multicollinearity, OLS estimators are BLUE.

b. In cases of high multicollinearity, it is not possible to assess the individual significance of one or more partial regression coefficients.

c. If an auxiliary regression shows that a particular R2 is high, there is definite evidence of high collinearity.

d. High pair-wise correlations do not suggest that there is high multicollinearity.

e. Multicollinearity is harmless if the objective of the analysis is prediction only.

f. Ceteris paribus, the higher the VIF is, the larger the variances of OLS estimators.

g. The tolerance (TOL) is a better measure of multicollinearity than the VIF.

h. You will not obtain a high R2 value in a multiple regression if all the partial slope coefficients are individually statistically insignificant on the basis of the usual t test.

i. In the regression of Y on X2 and X3, suppose there is little variability in the values of X3. This would increase var ($\hat{}$ β3). In the extreme, if all
X3 are identical, var ($\hat{}$ β3) is infinite.

Q.5 a. Show that if r1i = 0 for i = 2, 3, . . . , k then R1.2 3. . . k = 0

b. What is the importance of this finding for the regression of variable X1(=Y) on X2, X3, . . . , Xk?

3.7 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

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5. Gujarati, D.N.(1995). Basic Econometrics. McGraw Hill, New Delhi.

2.1 INTRODUCTION:

There are generally three types of data that are available for empirical analysis: (1) cross section, (2) time series, and (3) combination of cross section and time series, also known as pooled data. In developing the classical linear regression model (CLRM) we made several assumptions. However, we noted that *not* all these assumptions would hold in every type of data. As a matter of fact, we saw in the previous lesson that the assumption of homoscedasticity, or equal error variance, may not be always tenable in cross-sectional data. In other words, cross-sectional data are often plagued by the problem of heteroscedasticity.

However, in cross-section studies, data are often collected on the basis of a random sample of cross-sectional units, such as households (in a consumption

function analysis) or firms (in an investment study analysis) so that there is no prior reason to believe that the error term pertaining to one household or a firm is correlated with the error term of another household or firm. If by chance such a correlation is observed in cross-sectional units, it is called **spatial autocorrelation,** that is, correlation in space rather than over time. However, it is important to remember that, in cross-sectional analysis, the ordering of the data must have some logic, or economic interest, to make sense of any determination of whether (spatial) autocorrelation is present or not. The situation, however, is likely to be very different if we are dealing with time series data, for the observations in such data follow a natural ordering over time so that successive observations are likely to exhibit intercorrelations, especially if the time interval between successive observations is short, such as a day, a week, or a month rather than a year. If you observe stock price indexes, such as the Dow Jones or S&P 500 over successive days, it is not unusual to find that these indexes move up or down for several days in succession. Obviously, in situations like this, the assumption of **no auto,** or **serial, correlation** in the error terms that underlies the CLRM will be violated.

In this lesson we take a critical look at this assumption with a view to answering the following questions:

1. What is the nature of autocorrelation?

2. What are the theoretical and practical consequences of autocorrelation?

3. Since the assumption of no autocorrelation relates to the unobservable disturbances *ut*, how does one know that there is autocorrelation in any given situation? Notice that we now use the subscript *t* to emphasize that we are dealing with time series data.

4. How does one remedy the problem of autocorrelation?

2.2 OBJECTIVES:

1. Understand the meaning of autocorrelation.

2. Understand the consequences of autocorrelation on OLS estimates.

3. Detect autocorrelation through graph inspection.

4. Detect autocorrelation through formal econometric tests.

5. Distinguish among the wide range of available tests for detecting autocorrelation..

2.3 WHAT IS AUTOCORRELATION

Correlation between members of series of observation ordered in time (as in time series data) or space as in cross-sectional data)

Auto doesn't exist in the disturbance u_1)

$\Sigma(u_i u_j) = 0 \qquad i \neq j$

2.3.3 NATURE OF AUTOCORRELATION:

1. **Inertia**: - Silent feature of most of the time series is inertia or sluggishness. Well known, time series such as GNI price Index.

2. **Specification Bias: Excluded variable case**: - Residuals (which are proxies of u_i) may suggest that same variable that were originally candidates but were not included in the model for a variety of reasons should be included.

$$Y_i = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + u_i$$

Y = Quantity of beef demanded.

X_2 = Price of beef

X_3 = Consumer income

X_4 = Price of Pork

t = Time

AFTER REGRESSION:-

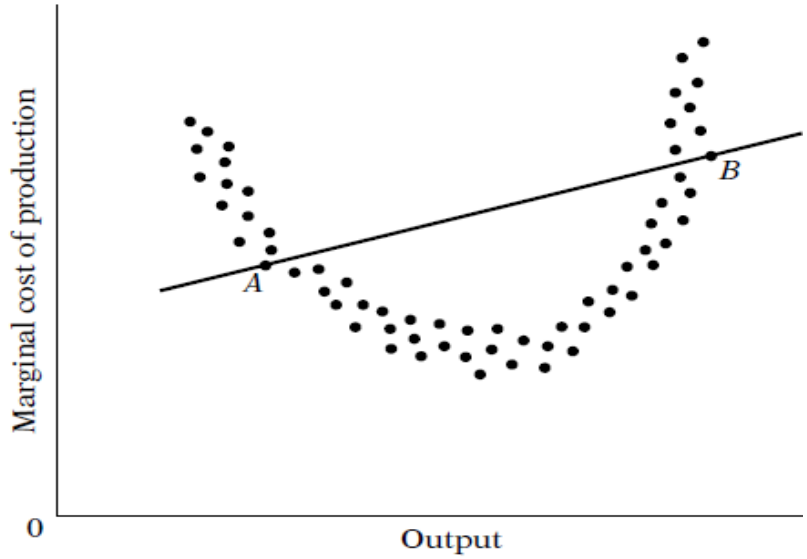
$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + V_t$$

3. **Specification Bias: Incorrect functional form**:-

$$\text{Marginal Cost}_t = \beta_1 + \beta_2 \,\text{output} + \beta_3 \text{output}_i^2 + u_i$$

But we get the following model.

$$MC_t = \alpha_1 + \alpha_2 \,\text{output}_t + V_i$$



MC curve corresponding to the 'true' model is along with the "incorrect" linear cost curve.

Specification bias: incorrect functional form.

4. **Cobweb Phenomenon**: - The supply of many agricultural commodities reflects the so called cobweb Phenomenon. Where supply reacts to price with a lag of one time period because supply decisions takes time implement.

$$Supply_t = \beta_1 + \beta_2 P_{t-1} + u_t$$

5. **Lag**: -

$$Consumption: - \beta_1 + \beta_2 Income + \beta_2 Consumption_{t-1} + u_t$$

6. **Manipulation of data**: - In empirical analysis the raw data are often manipulated.

7. **Data Transformation**:-

$Y_t = \beta_1 + \beta_2 X_t + u_t \quad \rightarrow 1$

Y = Consumption, X = Income

$Y_{(t-1)} = \beta_1 + \beta_2 X_{(t-1)} + u_{(t-1)}$ $\rightarrow 2$ Previous Period

$Y_{(t-1)}$, $X_{(t-1)}$, $u_{(t-1)}$ are lagged values of X_1 Y & U

Sub. (II) from (I) we get

$\Delta Y_t = \beta_2 \Delta X_t + \Delta u_t$ $\rightarrow$ Δ first difference operator

FOR EMPIRICAL PURPOSE

$\Delta Y_t = \beta_2 \Delta X_t + V_t$ $\rightarrow$ $V_t = \Delta u_t = (u_t - u_{t-1})$

2.3.4 TEST OF AUTOCORRELATION:

2.3.2.1 **Graphical Method**:-

- Plot any of error
- Error term & there exists non-stationary

Stationary

$Y_t = \rho Y_{t-1} + u_t$

$Y_t = Y_{t-1} + u_t$ $(\rho=1)$

$Y_t - Y_{t-1} = u_t$

Now assume there is lag operation (L)

$(Ly_t = Y_{t-1})$

$Y_t - LY_t = U_t$

$y_t (1-L) = U_t$

if (1-L) = 0

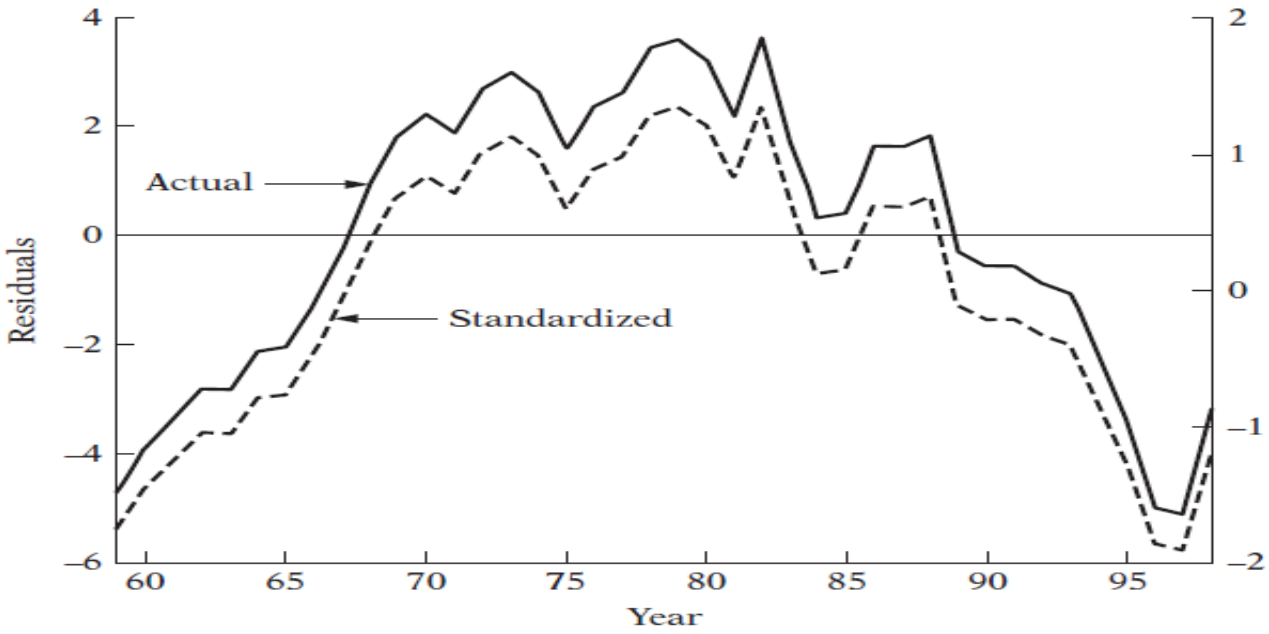
L = 1

This is known as unit root.

(When root is unit autocorrelation is there) (Non stationary & unit rest is same)

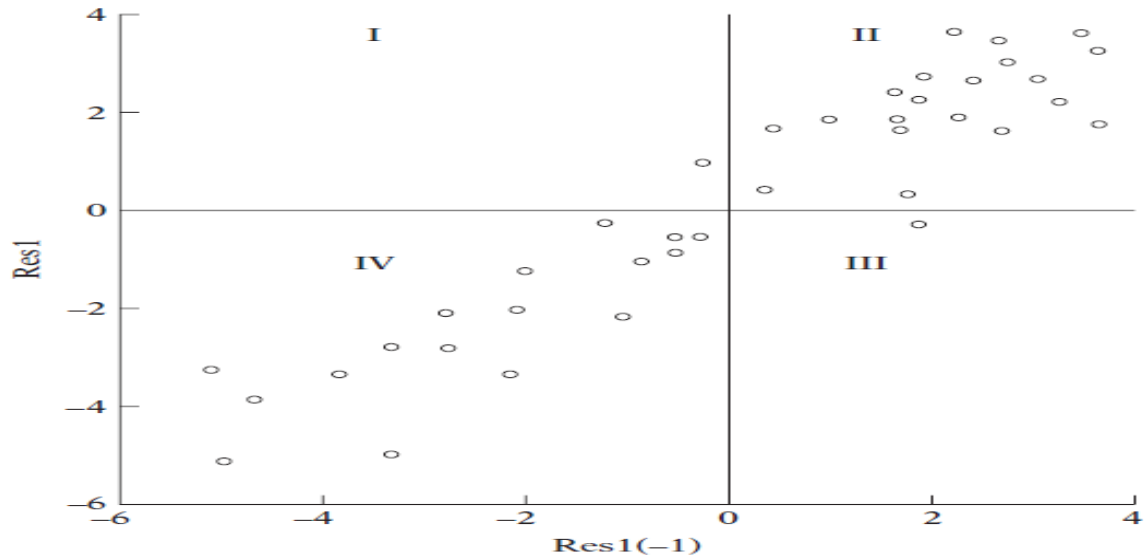
There are various ways of examine the residuals (error)

a) Time sequence plot



Residuals and standardized residuals from the wages–productivity regression (

b) Standardized residual



Current residuals versus lagged residuals.

Type your

2.3.2.2 **The Runs Test**:-

Initially, we have several residuals that are negative, then there is a series of positive residuals, and then there are several residuals that are negative. If these residuals were purely random, could we observe such a pattern? Intuitively, it seems unlikely. This intuition can be checked by the so-called runs test, sometimes also know as the Geary test, a nonparametric test.

(---------)(++++++++++++++++++++)(----------)

This is also a crude method.

We now define a run as an uninterrupted sequence of one symbol or attribute, such as + or -. We further define the length of a run as the number of elements in it.

2.3.2.3 **Durbin Watson test**:-

→ Also known as Durbin Watson *d* Test.

→ One of the good methods as the *d* statistic is based on the estimated residuals, which are computed in regression analysis

$$d = \frac{\sum(\hat{u}_t - \hat{u}_{t-1})^2}{\sum(\hat{u}_t)^2}$$

This tells where there exists autocorrelation or not

$$\frac{\sum \hat{u}_t^2}{\sum \hat{u}_t^2} + \frac{\sum \hat{u}_{t-1}^2}{\sum \hat{u}_t^2} - \frac{2\sum \hat{u}_t + \hat{u}_{t-1}}{\sum \hat{u}_t^2}$$

$\simeq$ 1 + 1 (by nearly) - $\frac{2\sum \hat{u}_t - \hat{u}_{t-1}}{\sum \hat{u}_t}$

$\simeq$ 2 (1 - $\frac{\sum \hat{u}_t - \hat{u}_{t-1}}{\sum \hat{u}_t}$)

d $\simeq$ $2(1-\hat{\rho})$

- 2 (1- (-1) = 4
- 2 (1 - (1) = 0

d will be $0 \leq d \leq 4$

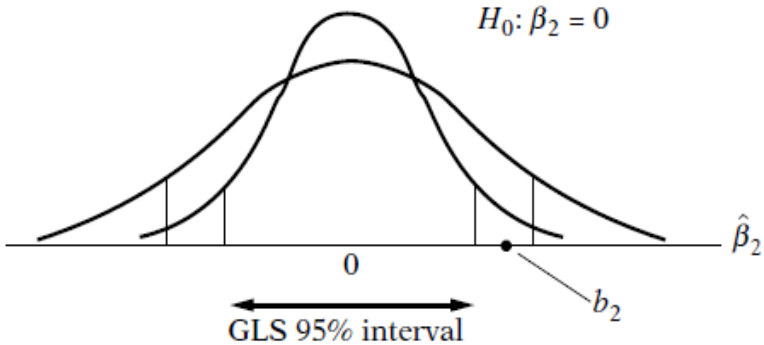
because $\rho = -1 \leq \rho \leq 1$

→ d $\simeq$ 2 → no autocorrelation

→ d $\simeq$ 0 or 4 (closer) there is autocorrelation

2.3.5 CONSEQUENCES OF AUTOCORRELATION:

2.3.3.1 OLS Estimation allowing for Autocorrelation.



GLS and OLS 95% confidence intervals.

To establish confidence interval to test hypotheses, one should be GLS & not OLS even though the estimators derived from the latter are unbiased & consistent.

2.3.3.2 Estimation Disregarding Autocorrelation.

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{(n-2)}$$

Unbiased estimate of σ^2 i.e $\sum(\hat{\sigma}_i^2) = \sigma^2$

$$\sum(\hat{\sigma}^2 - \frac{\sigma^2\{n-[2/(1-\rho]\}-2\rho r}{n-2}$$

2.3.6 REMEDIAL MEASURES OF AUTOCORRELATION:

1. Try to find out if the autocorrelation is pure autocorrelation or not because of the result of the mis-specification of the model.
2. Transformation of original model, so that in the transformed model we do not have the problem of (Pure) autocorrelation.
3. In case of large sample we can Newey-West method to obtain standard error of OLS estimators that are corrected for auto correlation.
4. In some situation we can continue to use the OLS method.

2.4 SUMMARY AND CONCLUSIONS:

1. If the assumption of the classical linear regression model—that the errors or disturbances *ut* entering into the population regression function (PRF) are random or uncorrelated—is violated, the problem of serial or autocorrelation arises.

2. Autocorrelation can arise for several reasons, such as inertia or sluggishness of economic time series, specification bias resulting from excluding important variables from the model or using incorrect functional form, the cobweb phenomenon, data massaging, and data transformation.

3. Although in the presence of autocorrelation the OLS estimators remain unbiased, consistent, and asymptotically normally distributed, they are no longer efficient. As a consequence, the usual *t*, *F*, and $\chi 2$ tests cannot be legitimately applied. Hence, remedial results may be called for.

4. The remedy depends on the nature of the interdependence among the disturbances *ut*. But since the disturbances are unobservable, the common practice is to assume that they are generated by some mechanism.

2.5 LETS SUM IT UP:

In last, we can say that this lesson in many ways similar to the preceding lesson on heteroscedasticity in that under both heteroscedasticity and autocorrelation the usual OLS estimators, although linear, unbiased, and asymptotically (i.e., in large samples) normally distributed, are no longer minimum variance among all linear unbiased estimators. In short, they are not efficient relative to other linear and unbiased estimators. Put differently, they may not be BLUE. As a result, the usual, t, F, and $\chi 2$ may not be valid.

2.6 EXCERCISES:

Q.1 State whether the following statements are true or false. Briefly justify your answer.

a. When autocorrelation is present, OLS estimators are biased as well as inefficient.
b. The Durbin–Watson d test assumes that the variance of the error term ut is homoscedastic.
c. The first-difference transformation to eliminate autocorrelation assumes that the coefficient of autocorrelation ρ is -1.
d. The R2 values of two models, one involving regression in the first difference form and another in the level form, are not directly comparable.
e. A significant Durbin–Watson d does not necessarily mean there is autocorrelation of the first order.
f. In the presence of autocorrelation, the conventionally computed variance and standard errors of forecast values are inefficient.
g. The exclusion of an important variable(s) from a regression model may give a significant d value.

Q.2 Given a sample of 50 observations and 4 explanatory variables, what can you say about autocorrelation if (a) d = 1.05? (b) d = 1.40? (c) d = 2.50?
(d) d = 3.97?

Q.3 In a sequence of 17 residuals, 11 positive and 6 negative, the number of runs was 3. Is there evidence of autocorrelation? Would the answer change if there were 14 runs?

Q.4 Explain the Durbin-Watson and Runs Test for detecting autocorrelation?

Q.5 Elaborate the various remedial measures of autocorrelation?

2.7 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

4. Green, W.(2000). Econometrics, Prentice Hall of India, New Delhi.

5. Gujarati, D.N.(1995). Basic Econometrics. McGraw Hill, New Delhi.

6. Koutsoyiannis,A.(1977). Theory of Econometrics(2nd Esdn.). The Macmillan Press Ltd. London.

7. Maddala, G.S.(1997). Econometrics, McGraw Hill; New York.

4.1 INTRODUCTION:

Let us return to our wages productivity regression. There we saw that the d value was 0.1229 and based on the Durbin-Watson d test we concluded that there was positive correlation in the error term. Could this correlation have arisen because our model was not correctly specified? Since the data underlying regression is time series data, it is quite possible that both wages and productivity exhibit trends. If that is the case, then we need to include the time or trend, t, variable in the model to see the relationship between wages and productivity net of the trends in the two variables.

To test this, we included the trend variable and obtained the following results.

$$\hat{Y}_t = 1.4752 + 1.3057X_t - 0.9032t$$
$$se = (1.318) \quad (0.2765) \quad (0.4203)$$
$$t = (0.1118) \quad (4.7230) \quad (-2.1490)$$
$$R^2 = 0.9631 \qquad d = 0.204$$

The interpretation of this model is straightforward: Over time, the index of real wages has been decreasing by about 0.90 units per year. After allowing for this if the productivity index went up by a unit, on average, the real wage index went up by about 1.30 units, although this number is not statistically different from one (why?). What is interesting to note is that even allowing for the trend variable, the d value is still very low, suggesting pure autocorrelation and not necessarily specification error.

To test this, we regress Y on X and X^2 to test for the possibility that the real wage index may be nonlinearly related to the productivity index. The results of this regression are as follows:

$$\hat{Y}_t = -16.2181 + 11.9488X_t - 0.0078X_t^2$$
$$t = (-5.4891) \quad (24.9868) \quad (-15.9363)$$
$$R^2 = 0.9947 \qquad d = 1.07$$

These results are interesting. All the coefficients are statistically highly significant, the p values being extremely small. From the negative quadratic term, it seems that although the real wage index increases as the productivity index increases, it increases at a decreasing rate. But look at the d value. It still suggests positive autocorrelation in the residuals, for $d_L = 1.391$ and $d_U = 1.60$ and the estimated d value lies below d_L.

It may be safe to conclude from the proceeding analysis that our wages-productivity regression probably suffers from pure autocorrelation and not necessarily from specification bias. Knowing the consequences of autocorrelation, we may therefore want to take some corrective action. We will do so shortly.

Incidentally, for all the wages productivity regression that we have presented above, we applied the Jarque–Bera test of normality and found that the residuals were normally distributed, which is comforting because the d terms assumes normally of the error term.

4.2 OBJECTIVES:

1. The key objective is to find what are the criteria in choosing a model for empirical analysis.

2. Our objective is to find what types of model mis- specification errors is one likely to encounter in practice.

3. The another objective is to find how does one evaluate the performance of competing models?

4.3 CORRECTING FOR (PURE) AUTOCORRELATION:

4.3.1 THE METHOD OF GENERALIZED LEAST SQUARES (GLS):

Knowing the consequences of autocorrelation, especially the lack of efficiency of OLS estimators, we may need to remedy the problem. The remedy depends on the knowledge one has about the nature of interdependence among the disturbances, that is, knowledge about the structure of autocorrelation.

As a starter, consider the two-variable regression model:

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

And assume that the error term follows the AR(1) scheme, namely,

$$(u_t - p u_{t-1}) = \varepsilon_t \quad -1 < p < 1$$

Now we consider two cases: (1) p is known and (2) ρ is not known but has to be estimated.

When ρ is known

If the coefficient of first-order autocorrelation is known, the problem of autocorrelation can be easily solved. Hence,

$$Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1} \quad 1$$

Multiplying by ρ on both sides, we obtain

$$\rho Y_{t-1} = \rho\beta_1 + \rho\beta_2 X_{t-1} + p u_{t-1} \quad 2$$

Subtracting (2) from (1) gives

$$(Y_t - \rho Y_{t-1}) = \beta_1(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + \varepsilon_t \quad 3$$

Where $\varepsilon_t = (u_t - p u_{t-1})$

We can express (3) as

$$Y_t^* = \beta_1^* + \beta_2^* X_t^* + \varepsilon_t \quad 4$$

Where $\beta_1^* = \beta_1(1-\rho), Y_t^* = (Y_t - \rho Y_{t-1}), X_t^* = (X_t - p X_{t-1}), and \beta_2^* = \beta_2$ 5

Since the error term in (4) satisfies the usual OLS assumptions, we can apply OLS to the transformed variables Y* and X* and obtain estimators with all the optimum properties, namely, BLUE. In effect, running is tantamount to using generalized least squares (GLS) discussed in the previous lesson – recall that GLS is nothing but OLS applied to the transformed model that satisfies the classical assumptions.

Regression (4) is known as the generalized, or quasi, difference equation. It involves regressing Y on X, not in the original form, but in the difference form, which is obtained by subtracting a proportion $(=\rho)$ of the value of a variable in the previous time period from its value in the current time period. In this differencing procedure we lose one observation because the first observation has no antecedent. To avoid this loss of one observation, the first observation on Y and X is transformed as follows. $Y_1\sqrt{1-p^2}\ and\ X_1\sqrt{1-p^2}.$ This transformation is known as the Prais-Winsten transformation.

4.3.2 OLS VERSUS FGLS AND HAC

The practical problem facing the researcher is this: In the presence of auto-correlation, OLS estimators, although unbiased, consistent, and asymptotically normally distributed, are not efficient. Therefore, the usual inference procedure based on the t, F, and χ^2 tests is no longer appropriate. On the other hand, FGLS (Feasible GLS and EGLS: Estimated GLS) HAC (Heteroscedasticity and autocorrelation estimation) produce estimators that are efficient, but the finite, or small-sample, properties of these estimators are not well documented. This means in small samples the FGLS and HAC might actually do worse than OLS. As a matter of fact, in a Monte Carlo study Griliches and Rao found that if the sample is relatively small and the coefficient of auto-correlation, ρ, is less than 0.3, OLS is as good or better than FGLS. As a practical matter, then, one may use OLS in small samples in which the estimated rho is, say, less than 0.3. Of course, what is a large and what is a small sample are relative questions, and one has to use some practical judgement. If you have only 15 to 20 observations, the sample may be small, but if you have, say, 50 or more observations, the sample may be reasonably large.

4.3.3 Coexistence of Autocorrelation and Heteroscedasticity

What happens if a regression model suffers from both heteroscedasticity and autocorrelation? Can we solve the problem sequentially, that is, take care of heteroscedasticity first and then autocorrelation? As a matter of fact, one author contends that "Autoregression can only be detected after the heteroscedasticity is controlled for". But can we develop an omnipotent test that can solve these and other problems (e.g., model specification) simultaneously? Yes, such tests exist, but their discussion will take us far afield. It is better to leave them for references.

4.4 SUMMARY AND CONCLUSIONS

1. If the assumption of the classical linear regression model that the errors or disturbances u_t entering into the population regression function (PRF) are random or uncorrelated – is violated, the problem of serial or autocorrelation arises.

2. Autocorrelation can arise for several reasons, such as inertia or sluggishness of economic time series, specification bias resulting from excluding important variables from the model or using incorrect functional form, the cobweb phenomenon, data massaging, and data transformation. As a result, it is useful to distinguish between pure autocorrelation and "induced" autocorrelation because of one or more factors just discussed.

3. Although in the presence of autocorrelation the OLS estimators remains unbiased, consistent, and asymptotically normally distributed, they are no longer efficient. As a consequence, the usual t, F, and χ^2 tests cannot be legitimately applied. Hence, remedial results may be called for.

4. The remedy depends on the nature of the interdependence among the disturbances u_t. But since the disturbances are unobservable, the common practice is to assume that they are generated by some mechanism.

5. The mechanism that is commonly assumed is the Markov first-order autoregressive scheme, which assumes that the disturbance in the current time period is linearly related to the disturbance term in the previous time period, the coefficient of autocorrelation p providing the extent of the interdependence. This mechanism is known as the AR(1) scheme.

6. If the AR(1) scheme is valid and the coefficient of autocorrelation is known, the serial correlation problem can be easily attacked by transforming the data following the generalized difference procedure. The AR(1) Scheme can be easily generalized to an AR(p). One can also assume a moving average (MA) mechanism or a mixture of AR and MA schemes, known as ARMA. This topic will be discussed in the lessons on time series econometrics.

7. Even if we use an AR(1) scheme, the coefficient of autocorrelation is not known a priori. We considered several methods of estimating p, such as the Durbin-Watson d, Theil-Nagar modified d, Cochrane-Orcutt (C-O) iterative procedure, C-O two step method, and the Durbin two-step procedure. In large samples, these methods generally yield similar estimates of p, although in small samples they perform differently. In practice, the C-O interative method has become quite popular.

8. Using any of the methods just discussed, we can use the generalized difference method to estimate the parameters of the transformed model by OLS, which essentially amounts to GLS. But since we estimate $\rho(=\hat{p}),$ we call the method of estimation as feasible, or estimated, GLS, or FGLS or EGLS for short.

1. In using EGLS, one has to be careful in dropping the first observation, for in small samples the inclusion or exclusion of the first observation can make a dramatic difference in the results. Therefore, in small samples it is advisable to transform the first observation according to the Prais-Winsten procedure. In large samples, however, it makes little difference if the first observation is included or not.

10. It is very important to note that the method of EGLS has the usual optimum statistical properties only in large samples. In small samples, OLS may actually do better that EGLS, especially if $\rho < 0.3$.

11. Instead of using EGLS, we can still use OLS but the correct the standard errors for autocorrelation by the Newey-West HAC procedure. Strictly speaking, this procedure is valid in large samples. One advantages of the HAC procedure is that it not only corrects for autocorrelation but also for heteroscedasticity, if it is present.

12. Of course, before remediation comes detection of autocorrelation. There are formal and informal methods of detection. Among the informal methods, once can simply plot the actual or standardized residuals, or plot current residuals against past residuals. Among formal methods, one can use the runs test, Durbin Watson *d* test, asymptotic normality test, Berenblutt-Webb test, and Breusch-Godfrey (BG) test. Of these, the most popular and routinely used is the Durbin-Watson *d* test, for it is much more general in that it allows for both AR and MA error structures as well as the presence of lagged regressed as an explanatory variable. But keep in mind that it is a large sample test.

4.5 LETS SUM IT UP:

In concluding remarks, we can say that if particular model is not specified correctly, we face the problem of model specification error or model specification bias.

4.6 EXCERCISES:

Q1 State Breusch Pagan Godfrey test.

Q2 What happens to OLS estimation in presence of autocorrelation?

Q3 What is EGLS or FGLS?

Q4 Does heteroscedasticity makes the estimators biased? Explain.

Q5 Describe correlation for pure autocorrelation.

Q6 Describe multicollinearity its test and remedial measures.

4.7 Suggested Reading / References:

1. Baltagi, B.H.(1998). Econometrics, Springer, New York.

2. Chow,G.C.(1983). Econometrics, McGraw Hill, New York.

3. Goldberger, A.S.(1998). Introductory Econometrics, Harvard University Press, Cambridge, Mass.

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UNIT - V

1.1 INTRODUCTION:

The classical linear regression model is that the disturbances *ui* appearing in the population regression function are homoscedastic; that is, they all have the same variance. In this lesson we examine the validity of this assumption and find out what happens if this assumption is not fulfilled. We seek answers to the following questions:

1. What is the nature of heteroscedasticity?
2. What are its consequences?
3. How does one detect it?
4. What are the remedial measures?

1.2 OBJECTIVES:

1. Understand the meaning of heteroskedasticity and homoskedasticity through examples.

2. Understand the consequences of heteroskedasticity on OLS estimates.

3. Detect heteroskedasticity through graph inspection.

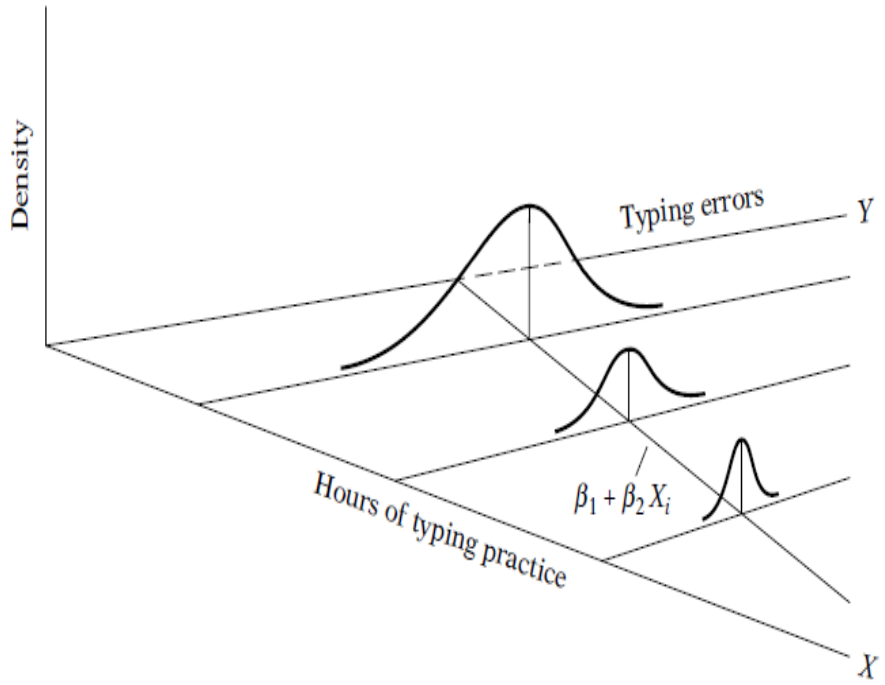
4. Detect heteroskedasticity through formal econometric tests.

5. Distinguish among the wide range of available tests for detecting heteroskedasticity.

1.3 HETEROSCEDASTICITY

Where the conditional variance of the Y population varies with X. This situation in known appropriately as heteroscedasticity or unequal spread or variance.

$$E(u_i^2)=\sigma_i^2$$



Higher income families on the arrange save more than the lower income family, but there is more variability in their savings

Illustration of heteroscedasticity.

1.3.2 Nature of Heteroscedasticity

2. It's an error learning model, as people learn, their error of behavoiur become smaller over time.
3. As income grow, people have more discretionary income & hence more scope for choice about the disposition of their income.
4. As data collecting techniques increases σ_i^2 is likely to decrease.
5. If can also arise as a result of the presence of collinear.
6. It is skewness in the distribution of one or more regressions included in the model.
7. Incorrect data transformation.
8. Incorrect functional form.

1.3.1.1 OLS Estimation in the Presence of Heteroscedasticity

$$E(u_i^2) = \sigma_i^2$$

$\therefore$ $Y_i = \beta_1 + \beta_2 X_i + u_i$

Applying the usual formula the OLS estimator is β_2 is

$$\beta 2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$$

$$= \frac{n\Sigma x_i y_i}{n\Sigma X_i} - \frac{\Sigma x_i y_i}{(\Sigma X_i)^2}$$

$$\therefore \quad Var\hat{\beta}_2 = \frac{\sigma_i^2}{\sum x_i^2}$$

$$\therefore \quad Var\hat{\beta}_2 = \frac{\sum f^2 \sigma_i^2}{(\sum x_i^2)^2}$$

$$\therefore \quad Var(\hat{\beta}_2) = \frac{\sigma_i^2}{\sum x_i^2}$$

1.3.3 DETECTION OR TEST

1.3.2.1 Informal Methods

1. **Nature of Problem: -** Very often nature of the problem under consideration suggests whether heteroscedasticity is likely to be encountered.

2. **Graphical Problem**: - If there is no empirical information about the nature of heteroscedasticity, in practice one can do the regression analysis on the assumption that there is no heteroscedasticity & then do a postmortem

examination of the residual squared $\hat{u}_i^2$ to see if they exhibit any systematic pattern.

1.3.3.2 Formal Methods

1. **Park Test**: - Park formalized the graphical method by suggesting that σ_i^2 is same function of the explanatory variable X_i.

   His suggested function was

$$\sigma_i^2 = \sigma^2 X_i^{\beta} e^{vi}$$
$$or$$
$$\ln\sigma_i^2 = \ln\sigma^2 + \beta \ln X_i + V_i$$

   Since σ_i^2 is generally not known. Park suggested using $\hat{u}_i$, as a proxy & running following regression.

$$\ln\hat{u}_i^2 = \ln\sigma^2 + \beta \ln X_i + V_i$$
$$= \alpha + \beta \ln X_i + V_i$$

   - If β turn out to be statistically significant, it would suggest that heteroscedasticity is present in the data.
   - Park test is two stage procedure
     a) We run the OLS regression disregarding the heteroscedasticity question.
     b) Run the regression.

2. **Glejser Test**: - It is as Park test. He suggests regressing the absolute values of $\hat{u}_i$ on the X variable.

$$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 X_i^2} + V_i$$

3. **Spearman's Rank Correlation Test**:-

$$r_s = 1 - 6\left[\frac{\sum d_i^2}{n(n^2-1)}\right]$$

d_i = difference in the rank

n= no. of individual.

4. **GoldFeld Quandt Test**: - One of the popular methods, in which of one assumes that the heteroscedasticity variance σ_i^2 is positively related to one of the explanatory valuables in the regression model.

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

Suppose σ_i^2 is positively related to X_i

$\sigma_i^2 = \sigma^2 X_i^2$

5. **Breusch Pagan Godfrey Test (BPG Test)**:-

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \ldots\ldots \beta_k X_k + u_i$$
$$\sigma_i^2 = f(\alpha_0 + \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 \ldots\ldots + \alpha_m z_m$$
$$\sigma_i^2 = \alpha_0 + \alpha_1 z_1 + \alpha_2 z_2 \ldots\ldots \alpha_m z_m$$

(Linear functions)

$$\alpha_0 = \alpha_1 = \alpha_2 \ldots\ldots \alpha_m = 0$$

{No heteroscedasticity, no relation between two}

Run the regression

$$\hat{u}_1^2 = \alpha_0 + \alpha_1 z_1 + \alpha_2 z_2 \ldots\ldots \alpha_m z_m$$

$\theta = \frac{1}{2}$ ESS

6. **White Test**: - (Most logical for all)

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u_2$$

Error may be related between X_1 & X_2.

$$\hat{u}_1^2 = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_1^2 + + \alpha_4 X_2^2 + \alpha_5 X_1 X_2 + v_i$$

of $\alpha_0 = \alpha_1 = \alpha_2 \ldots\ldots \alpha_n = 0$ (No heteroscedasticity)

White test can be a test of heteroscedasticity or specification error or both.

1.3.3 CONSEQUENCES OF USING OLS IN THE PRESENCE OF HETEROSCEDASTICITY

As we have seen, both $\hat{\beta}_2^*$ and $\hat{\beta}_2$ are (linear) unbiased estimators: In repeated sampling, on the average, $\hat{\beta}_2^*$ and $\hat{\beta}_2$ will equal the true β_2; that is, they are both unbiased estimators. But we know that it is $\hat{\beta}_2^*$ that is efficient that is, has the smallest variance. What happens to our confidence interval, hypotheses testing, and other procedures if we continue to use the OLS estimator $\hat{\beta}_2^*$? We distinguish two cases.

1.3.3.1 OLS Estimation allowing for heteroscedasticity

Suppose we use $\hat{\beta}_2^*$ and use the variance formula given in var ($\hat{\beta}_2$) = $\frac{\Sigma x_1^2 \sigma_1^2}{(\Sigma x_1^2)^2}$, which takes into account heteroscedasticity explicitly. Using this variance, and assuming σ_i^2 are known, can we establish confidence intervals and test hypotheses with the usual *t* and *F* test? The answer generally is no because it can be shown that var($\hat{\beta}_2^*$) < var($\hat{\beta}_2^*$

),[5] which means that confidence intervals based on the latter will be unnecessarily larger. As a result, the *t* and *F* test are likely to give us inaccurate results in that var ($\hat{\beta}_2$)is overly large and what appears to be a statistically insignificant coefficient (because the t value is smaller than what is appropriate) may in fact be significant if the correct confidence intervals were established on the basis of the GLS procedure.

1.3.3.2 OLS Estimation disregarding heteroscedasticity

The situation can become serious if we not only use $\hat{\beta}_2$ but also continue to use the usual (Homoscedasticity) variance formula given in var ($\hat{\beta}_2$)= $\frac{\sigma^2}{\Sigma x_1^2}$ even if heteroscedasticity is present or suspected: Note that this is the more likely case of the two we discuss here running in standard OLS regression package and ignoring)or being ignorant (or being ignorant of) heteroscedasticity will yield variance of $\hat{\beta}_2$. First of all car ($\hat{\beta}_2$)is a biased estimator of var ($\hat{\beta}_2$) that is, on the average it over estimates or underestimates the latter, and in general we cannot tell whether the bias is positive (overestimation) or negative (underestimation) because it depends on the nature of the relationship between $\sigma_i^2$and the values taken by the explanatory variable X,. The bias arise from the fact that $\hat{\sigma}^2$, the conventional estimator of $\hat{\sigma}^2$, namely $\Sigma \hat{u}_i^2$(n-2) is no longer an unbiased estimator of the latter when heteroscedasticity in present . As a result, we can no longer rely on the conventionally computed confidence intervals and the conventionally employed *t* and F tests. In short, if we persist in using the usual testing procedures despite heteroscedasticity, whatever conclusions we draw or inferences we make may be very misleading.

To throw more light on this topic, we refer to a Monte Carlo study conducted by Davidson and MacKonnon. They consider the following simple model, which in our notation is

$Y_i = \beta_1 + \beta_2 X_i + u_i$

They assume that $\beta_1 = 1$, $\beta_2 = 1$, and $u_i \sim N(0, X_i^{\alpha})$.

From the preceding discussion it is clear that heteroscedasticity is potentially a serious problem and the researcher needs to know whether it is present in a given situation. If its presence is detected, then one can take corrective action, such as using the weighted least-squares regression or some other technique. Before we turn to examining the various corrective procedures, however, we must first find out whether the various corrective procedures, however, we must first find out whether heteroscedasticity is present or likely to be present in a given case.

1.3.4 REMEDIAL MEASURES

When σ_i^2 is known: The method of weighted least squares

As we have seen, if σ_i^2 is known, the most straight forward method of correcting heteroscedasticity is by means of weighted least squares, for the estimators thus obtained are BLUE.

When σ_i^2 is not known

If true σ_i^2 are known, we can use the WLS method to obtain BLUE estimators. Since the true σ_i^2 are rarely known, is there a way of obtaining consistent (in the statistical sense) estimates of the variances and co-variances of OLS estimators even if there is heteroscedasticity? The answer is yes.

White's Heteroscedasticity-Consistent Variances and Standard Errors. White has shown that this estimate can be performed so that asymptotically valid (i.e., large-

sample) statistical inferences can be made about the true parameter values. We will not present the mathematical details, for they are beyond the scope of this book. Nowadays, several computer package present White's heteroscedasticity-corrected variances and standard errors along with the usual OLS variances and standard errors. Incidentally, White's heteroscedasticity corrected standard errors are also known as robust standard errors.

1.4 SUMMARY AND CONCLUSIONS:

1. A critical assumption of the classical linear regression model is that the disturbances *ui* have all the same variance, $\sigma 2$. If this assumption is not satisfied, there is heteroscedasticity.

2. Heteroscedasticity does not destroy the unbiasedness and consistency properties of OLS estimators.

3. But these estimators are no longer minimum variance or efficient. That is, they are not BLUE.

4. The BLUE estimators are provided by the method of weighted least squares, provided the heteroscedastic error variances, $\sigma 2\ i$, are known.

5. In the presence of heteroscedasticity, the variances of OLS estimators are not provided by the usual OLS formulas. But if we persist in using the usual OLS formulas, the *t* and *F* tests based on them can be highly misleading, resulting in erroneous conclusions.

6. Documenting the consequences of heteroscedasticity is easier than detecting it. There are several diagnostic tests available, but one cannot tell for sure which will work in a given situation.

7. Even if heteroscedasticity is suspected and detected, it is not easy to correct the problem. If the sample is large, one can obtain White's heteroscedasticity corrected standard errors of OLS estimators and conduct statistical inference based on these standard errors.

8. Otherwise, on the basis of OLS residuals, one can make educated guesses of the likely pattern of heteroscedasticity and transform the original data in such a way that in the transformed data there is no heteroscedasticity.

1.5 LETS SUM IT UP:

In the conclusion we can say that in the classical linear regression model the assumption that the disturbances *ui* have all the same variance, $\sigma 2$, if this assumption is not satisfied, we face the problem of heteroscedasticity.

1.6 EXCERCISES:

Q. 1 State with brief reason whether the following statements are true, false,or uncertain:

a. In the presence of heteroscedasticity OLS estimators are biased as well as inefficient.

b. If heteroscedasticity is present, the conventional t and F tests are invalid.

c. In the presence of heteroscedasticity the usual OLS method always overestimates the standard errors of estimators.

d. If residuals estimated from an OLS regression exhibit a systematic pattern, it means heteroscedasticity is present in the data.

e. There is no general test of heteroscedasticity that is free of any assumption about which variable the error term is correlated with.

f. If a regression model is mis-specified (e.g., an important variable is omitted), the OLS residuals will show a distinct pattern.

g. If a regressor that has nonconstant variance is (incorrectly) omitted from a model, the (OLS) residuals will be heteroscedastic.

Q. 2 What do mean by heteroskedasticity?

Q. 3 What are the formal and informal methods of detecting heteroscedasticity?

Q.4 What are the remedial measures which we can take in case of heteroscedastucity?

Q.5 Explain the OLS estimation in the presence of Heteroscedasticity ?

1.7 Suggested Reading / References:

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